

# Capital investments and price agreements in semicollusive markets

Chaim Fershtman\*

and

Eitan Muller\*

*We consider a semicollusive market where firms compete in a long-run variable, such as investment in capital or capacity, and collude with respect to a short-run variable, such as price or market shares. Our concern is with the potential destabilizing effect of the long-run competition on the short-run collusion. We show that under a certain refinement of the equilibrium, the set of equilibria is reduced to include just the one in which the collusive agreement is stable. We then lend some support to the phenomenon of an inverse association between advertising and competition by investigating the conditions under which overcapitalization occurs in the above equilibrium.*

## 1. Introduction

■ The main purpose of this article is to examine behavior in “semicollusive” markets, where rivals compete in one variable (or set of variables) and collude in another. Scherer (1970), for example, notes that firms in oligopolistic markets often tend not to compete with respect to prices, but rather to compete in nonprice variables, such as technological innovation, advertising, and product differentiation. The common element in these nonprice variables is that they all involve investment over time. When the cost of investment is convex (as we assume), the changes in capital stocks are not instantaneous. Moreover, in many instances investment decisions must be made far in advance so that they are observed only with a time lag. As a result, a firm will be reluctant to enter an agreement on its capital stock, since a breakdown in the agreement may leave the firm either overcapitalized or, even worse, undercapitalized with a weak market position. By contrast, firms can more easily adjust their behavior if an agreement on pricing breaks down.

Our main results are as follows. First, we prove the existence of an equilibrium in which firms compete in long-run investment variables and collude with respect to short-run variables, such as prices and market shares. Second, we show that, if we restrict the strategy space of each player unilaterally to be consistent with finding an actual best response,

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\* The Hebrew University of Jerusalem.

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the set of equilibria is reduced to include only the stable equilibrium. Third, we lend some support to the view that advertising and competition are inversely related by investigating the conditions under which overcapitalization occurs.

Specifically, consider firms as accumulating capital according to, say, the Nerlove–Arrow (1962) accumulation equation. In each period price and market shares are determined through a (cooperative) bargaining process. Although in each period firms may cooperate and divide the market among them, they realize that their relative bargaining power depends on the capital stock that they hold. This induces competitive behavior in investments, even where firms find it optimal to collude with respect to prices and market shares.

Our work, especially in the role we assign to capital, is different from much of the recent work on collusive or semicollusive environments. For example, Brock and Scheinkman (1985) model an oligopoly with capacity constraints in a supergame structure. Their main emphasis is on the role of capacity constraints in enforcing a collusive agreement on prices. In our model capital enters the payoff function explicitly, the investment in capital is a strategic variable, and capital does not act as a capacity constraint. The term capital thus embodies more than productive capabilities and can take different forms, such as goodwill and technical skill.

To capture the dynamic aspect of the competition, we use the differential games framework. Spence (1979) has studied the problem of sequential entry in a new market where firms have financial constraints on their investment rates. In our model capital, in addition to its productive capabilities, acts as a *power base for the collusive agreement*. If the firms fail to reach an agreement on prices, they will find themselves in a noncooperative game that is similar to Spence's. There are some notable differences, though, as we assume a convex cost function and depreciable capital, but do not impose any financial constraints. In addition, the firms' strategy space in our setting allows the firms to observe the state of the competition in the market and to react accordingly.

The key issue is the stability of the collusive agreement over time. To state that a firm *might* find it optimal to collude is not sufficient. We wish to determine whether firms can collude on prices and market shares throughout the (infinite) planning horizon and whether competitive capital accumulation can destabilize such collusion.

Osborne (1976) and Porter (1981) have argued that cheating, and the difficulty in detecting deviations from an agreement, are two main sources of cartel instability. In particular, Porter assumed that firms observe only their own production and the market price, but not the quantity produced by other firms. If market demand has a stochastic element, cheating is difficult to detect, since an unreasonably low price can be a result of cheating (i.e., deviation from the agreed upon output levels) as well as a result of an abrupt decline in demand. Because our model is deterministic, the problem of detecting cheating does not arise, since if the price deviates from the agreed upon price or one firm fails to appear at the bargaining session, the collusion breaks down.

In our model there are two possible paths of capital investment for each firm: one for the case of collusion and the other for the case of a breakdown in the collusive agreement. Although each firm benefits from the collusive agreement on prices, in evaluating the total benefits from collusion each firm must take into account competition in the long-run variable. Any firm may find that its capital stock in equilibrium is such that the cost of maintaining it exceeds the benefits from collusion. If this is the case, the firm will break the agreement. If the equilibrium of the game is such that the collusive agreement remains in effect throughout the (infinite) horizon, we call such a solution a stable collusive equilibrium.

We show the existence of a stable collusive equilibrium for such a game. Our game, however, has the additional complexity of possessing multiple equilibria. We wish to determine whether the stable collusive equilibrium has some desirable properties that are not shared by other equilibria. We therefore restrict or “retract” the set of strategies of each player in such a way that if we propose to each player that he plays strategies from this

retract, he will agree and will have no incentive throughout the game to use any strategy that is not in the retract. What we show is that such a retract is exactly what characterizes stable collusive equilibrium.

Lastly, we investigate the possibility of overcapitalization that might occur in the collusive market. When the capital in question is goodwill, the phenomenon is heavily empirically researched; see, for example Comanor and Wilson (1979). Telser (1964) observes: “There is little empirical support for an inverse association between advertising and competition, despite some plausible theorizing to the contrary.” What we show is that even the theoretical support is rather weak. It highly depends on the structure of the benefits that the firm achieves while engaging in the collusive agreement.

## 2. The model

■ We investigate a market in which firms evaluate a collusive arrangement as follows: price and market shares will be decided upon as a result of a bargaining process among the participating firms at each period of time. Let  $M$  be the set of all possible allocations of market shares to firms. The result of the negotiation at time  $t$  will be a price  $p(t) \in R_+$  and market shares vector  $m(t) \in M$ . The firms do not divide the market equally among them because, when they enter the bargaining game, they view their relative bargaining powers as different since they have different levels of capital. The firms accumulate this capital, which might be goodwill or capacity, according to the following accumulation equation:

$$\dot{K}_i = I_i - \delta_i K_i, K_i(0) = K_{i0}, \quad i \in N, \tag{1}$$

where  $K_i(t)$  denotes the  $i$ th firm’s capital,  $\delta_i$  is the depreciation parameter, a dot above the variable denotes differentiation with respect to time,  $N$  is the set of firms, and  $I_i$  is the investment in capital, which is assumed to belong to a compact set  $[0, \bar{I}_i]$ .

Because additional capital increases a firm’s bargaining power, there is competition in investment, while at the same time, firms may agree on prices and market shares. The market can therefore be described as semicollusive. We assume that the firms are fully rational and fully informed about the effect of their capital levels on the bargaining game.

Since we focus on the relationship between the different dimensions of market behavior, we do not specify the bargaining process with respect to the short-run variables, but assume that a solution exists. For each possible outcome of the negotiation  $p(t)$  and  $m(t)$ , let  $\pi(K(t), p(t), m(t))$  denote the payoff vector that is the vector of gross operating profits, net of all costs except the cost of investment in capital. We can describe the set of all possible payoffs as

$$S(K(t)) = \{ \pi(K(t), p(t), m(t)) \in R_+^n \mid p(t) \in R_+, m(t) \in M \}.$$

Let  $\pi_r(K) \in R_+^n$  be the payoff vector if the firms fail to reach an agreement. In this case they resort to rivalry, and for our purpose it is sufficient to assume that an equilibrium will result. Clearly, the form of competition or the choice of strategic variables (quantity or price) affects the payoff vector at the equilibrium. We do not, however, specify the nature of this competition.<sup>1</sup>

Assuming that the solution of the bargaining process is given by  $\mu(S(K), d(K))$ , we let  $\Phi(K) \in R^n$  be the firms’ benefits from engaging in a collusive behavior at time  $t$ :

$$\Phi(K) = \mu(S(K), d(K)) - \pi_r(K), \tag{2}$$

where  $d(K) = \pi_r(K)$  is the payoff vector achieved under the rivalry equilibrium, i.e., the threat-point payoffs.<sup>2</sup> We assume that  $\pi_{ri}(K), \Phi_i(K) \in C^2$  are increasing concave functions

<sup>1</sup> See Spence (1979) for a similar approach.

<sup>2</sup> In solving the bargaining problem, we can distinguish two main approaches. In the axiomatic approach, first presented by Nash (1950), the problem can be characterized by two components  $(S, d)$ , where  $d$  is a point in

with bounded first and second derivatives of  $K_i$ , that they are decreasing in  $K_j$ , and that  $\partial^2 \pi_{ri} / \partial K_i \partial K_j \neq 0$ .

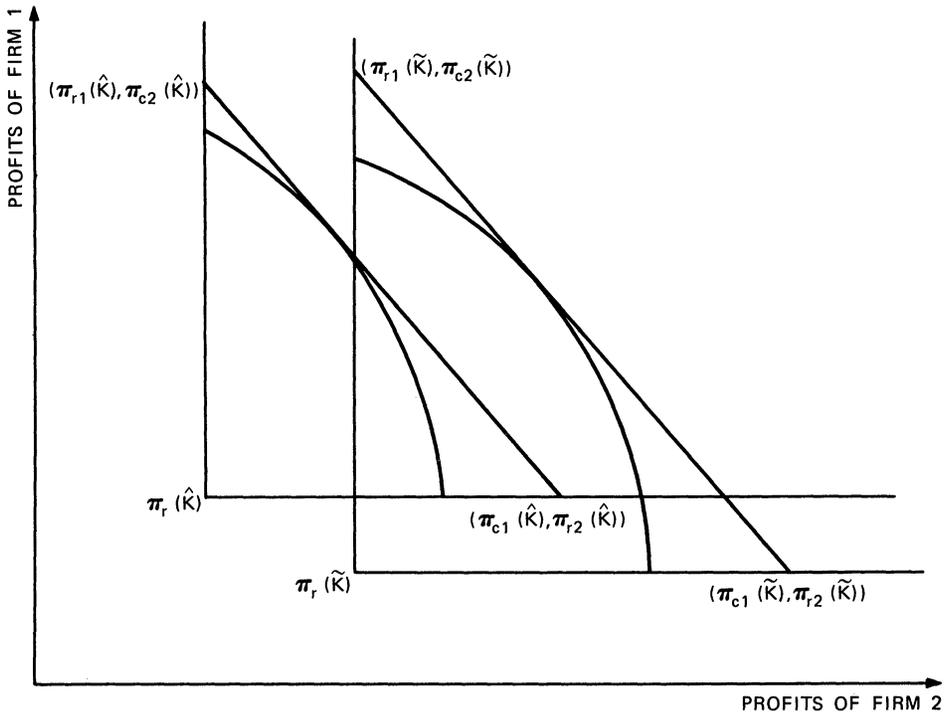
The individual rationality requirement guarantees that  $\Phi(K)$  is nonnegative. Each firm has to decide whether to stay within the collusive agreement. Once this is decided, each must choose an optimal path of investment in capital. We shall discuss the optimal strategy and the analysis of equilibrium after providing an illustrative example of bargaining with side payments.

### 3. An example: collusion with side payments

■ We now consider the model described in the previous section for the two-player case. In addition to the assumptions made there, assume the axiomatic bargaining approach and the possible existence of side payments. The bargaining set in this example is then a triangle as depicted in Figure 1.

Let  $\pi_r(K) = (\pi_{r1}(K), \pi_{r2}(K))$  be the threat point for a capital level of  $K$ . That is,  $\pi_{ri}$  is the payoff for firm  $i$  if the firms fail to reach an agreement. Let  $\pi_{ci}(K)$  be the maximum

FIGURE 1  
BARGAINING WITH SIDE PAYMENTS



$R^n$  that describes the outcome when no agreement is reached, and  $S$  is a compact convex subset of  $R^n$  containing  $d$ , which describes the set of all feasible (utility) payoffs that can be reached by cooperation. In the strategic approach the bargaining process is described without imposing any axioms on the solution functions. Examples of this approach are Rubinstein (1982), in which the bargaining process is described as a sequence of proposals, response, and counterproposals, and Crawford (1982), in which the bargaining process is formalized as a struggle between parties to commit themselves to favorable bargaining positions.

Since our main focus is the long-run competitive dimension and its possible destabilizing effect on collusion in prices and quantity, we choose not to specify the bargaining process itself, and instead use a general solution function that describes the gains from the collusive agreement as a function of the firms' capital.

profit firm  $i$  can enjoy when it receives all benefits from the collusive agreement. Finally, let  $\tilde{K}$  and  $\hat{K}$  be two arbitrary capital levels.

In Figure 1 the extreme points of the bargaining triangle are constructed such that all benefits from collusion will be accrued by firm 1 (respectively 2) to achieve the point  $(\pi_{c1}(\tilde{K}), \pi_{r2}(\tilde{K}))$  (respectively  $(\pi_{r1}(\hat{K}), \pi_{c2}(\hat{K}))$ ).

In this case we can use either the Kalai-Smorodinsky (1975) or the Nash solution to arrive at

$$\mu(S(K), d(K)) = ((\pi_{r1}(K) + \pi_{c1}(K))/2, (\pi_{r2}(K) + \pi_{c2}(K))/2). \tag{3}$$

Observe that both the extreme points and the threat point are functions of the capital levels  $K = (K_1, K_2)$ . In Figure 1 we have depicted two cases involving different amounts of capital,  $\tilde{K}$  and  $\hat{K}$ . Note that it is possible that  $\hat{K} > \tilde{K}$ , since we have assumed that  $\pi_{ri}(K)$  is increasing in  $K_i$  and decreasing in  $K_j$ . Therefore, the movement of the threat point and the extreme points depend on the magnitude of the changes in both levels of capital. The solution, therefore, changes as well, according to equation (3). Now it becomes clear that the collusive agreement is not costless, since the levels of capital must be supported by appropriate levels of investment. The profits  $\pi_r$  and  $\pi_r + \Phi$  are gross operating profits, net of all costs except investment in capital. Thus, it is not obvious *a priori* in which case the net profits are higher. In the rest of this article we do not assume necessarily that side payments exist, and so we return to our general formulation.

#### 4. The dynamic collusive game

■ To specify our main game we first deal with two simple games. Let  $C_i(I_i)$  be the investment cost of firm  $i$ . We assume that  $C_i(I_i) \in C^2$  is increasing and strictly convex, that  $C'_i$  is bounded from below, and that  $C'_i(0) = 0$ . In addition, we assume that  $I(t)$  takes its values in a compact set  $[0, \bar{I}]$ . For example, a cost function satisfying the condition that  $C(I)$  tends to infinity as  $I$  tends to  $\bar{I}$  will induce a control in the above compact set.

*Game A.* Let  $G_A(K_0)$  be the game in which the players resort to rivalry, i.e., it starts at the initial stocks of  $K_0$ , and the objective of each firm is to select investment path  $I_i(t)$  to maximize the payoff function:

$$J_{Ai} = \int_0^\infty e^{-rt} \{ \pi_{ri}(K) - C_i(I_i) \} dt, \quad i = 1, \dots, n. \tag{4}$$

*Game B.* Let  $G_B(K_0)$  be the game in which all players collude throughout the planning horizon, i.e., it starts at the initial stocks of  $K_0$ , and the objective of each firm is to select an investment path that maximizes the payoff function:

$$J_{Bi} = \int_0^\infty e^{-rt} \{ \pi_{ri}(K) + \Phi_i(K) - C_i(I_i) \} dt, \quad i = 1, \dots, n. \tag{5}$$

These two games belong to the class of capital accumulation games that we have discussed in previous works (Fershtman and Muller, 1984, 1986). Games *A* and *B* are formulated with an open-loop solution, although this is known to have some limitations (Kydlund, 1977; Spence, 1979). The closed-loop Nash equilibria, however, are known to exist only with severe limitations on the structure and duration of the game. For the above class of games, the closed-loop Nash equilibrium is not tractable.<sup>3</sup> When we later discuss the game in which firms are allowed to break the collusive agreement (game *C*), we shall depart from the open-loop formulations. Firms will be allowed to observe the state of the competition in the industry (rivalry or collusion) and to condition their behavior on this observation.

<sup>3</sup> For a discussion of tractable classes of differential games, see Case (1979, chap. 9).

Differential games allow us to overcome one main difficulty that arises in discrete time formulation. The difficulty, as noted by Maskin and Tirole (1983), is that in the latter formulation firms might wish to move just momentarily after a rival. This is of special interest in our analysis since a source of potential instability is the possible *momentary* (or transitional) gains a firm can make by breaking the agreement before its rivals find out and react. The continuous-time framework allows us to isolate other sources of instability.

*Definition 1.* A Nash equilibrium for the game  $G_A(K_0)$  (respectively  $G_B(K_0)$ ) is a vector of functions  $I^*(t)$  such that  $I_i^*(t)$  maximizes  $J_{Ai}$  (respectively  $J_{Bi}$ ) subject to (1), given  $(I_1^*(t), \dots, I_{i-1}^*(t), I_{i+1}^*(t), \dots, I_n^*(t))$ .

*Definition 2.* A stationary Nash equilibrium for  $G_A(K_0)$ , ( $G_B(K_0)$ ) is a vector of values  $(I^*, K^*)$  such that  $I_i^* = \delta_i K_i^*$ , and the vector  $I^*$  is a Nash equilibrium for the game  $G_A(K^*)$  (respectively  $G_B(K^*)$ ).

In Theorem 1 we state conditions for existence and convergence properties for games  $A$  and  $B$ . We shall use these to show the existence of a stable equilibrium for the game of interest in which the firms can choose the time for breaking the agreement and the alternative investment path used after doing so.

*Theorem 1.* Games  $A$  and  $B$  as defined in equations (4) and (5) satisfy the following. (a) For every initial capital stock  $K_0$ , there exists a Nash equilibrium solution. (b) If for game  $A$   $|\partial^2 \pi_{ri} / \partial K_i^2| > \sum_{j \neq i} |\partial^2 \pi_{ri} / \partial K_i \partial K_j|$  and for game  $B$

$$|\partial^2 \pi_{ri} / \partial K_i^2 + \partial^2 \Phi_i / \partial K_i^2| > \sum_{j \neq i} |\partial^2 \pi_{ri} / \partial K_i \partial K_j + \partial^2 \Phi_i / \partial K_i \partial K_j|,$$

then there exists a unique stationary Nash equilibrium point for each game. (c) Under the above conditions, from every initial capital stock  $K_0$ , every Nash equilibrium solution converges to the unique stationary equilibrium point.

*Proof.* The conditions of Theorem 1 follow the requirements of Theorems 2 and 1 in Fershtman and Muller (1984) and (1986), respectively. *Q.E.D.*

To see the economic intuition of condition (b) for the two-player case, assume that it does not hold, so that  $\partial^2 \pi_i / \partial K_i^2 < \partial^2 \pi_i / \partial K_i \partial K_j$ . The effects, therefore, of  $j$ 's action on  $i$ 's marginal profits are larger than the effects of  $i$ 's own actions. Any action of  $j$  will result in a larger reaction of the rival that causes a chain reaction that diverges rather than converges. Indeed, in the proof of Theorem 1 (1986) we have used exactly the "dampening" effect of condition (b) to show that such chain reactions become smaller and converge to zero as time approaches infinity.

Game  $A$  is the rivalry game in which the firms do not engage in any collusive agreements. In game  $B$ , however, the firms collude throughout the planning horizon *without considering the possibility of breaking the agreement*. Since consideration of breaking the agreement is a generic part of a reasonable economic game, we define game  $C$  to include such a possibility.

Let  $B \in \{1, 0\}$  be a variable that describes the state of competition in the industry. If at time  $t$  there is collusion with respect to price and market share, then  $B(t) = 1$ . If there is no collusion, then  $B(t) = 0$ . Firm  $i$ 's strategy for game  $C$  is assumed to belong to the following set:

$$\Omega_{ci} = \{T_i, I_i(t), B(t): [0, \infty) \times B \rightarrow [0, \bar{I}_i] | T_i \in [0, \infty),$$

$$I_i \text{ is piecewise continuous on } [0, \infty)\},$$

where  $T_i$  is defined as the time at which firm  $i$  decides to break the collusive agreement. If the collusive agreement is broken, each firm will choose an investment path that differs from the one that would have been followed had collusion continued.

The game is thus defined as follows.

*Game C.* Let  $G_c(K_0)$  be the game that starts at the initial stocks of  $K_0$ , with strategy space  $\Omega_c$ , equation (1), and payoff functions as follows:

$$J_{ci} = \int_0^\infty e^{-rt} \{ \pi_{ri}(K) + B(t)\Phi_i(K) - C_i(I_i) \} dt. \tag{6}$$

### 5. Stable collusion

■ What are the pitfalls of the collusive agreement we have just described? In a repeated Cournot oligopolistic game, one player may find it advantageous not to cooperate if the momentary (or transitional) gains he makes by not cooperating dominate the discounted losses he makes when all other players stop cooperating. Moreover, it has been argued that cheating might be hard to detect in a cartel environment. To quote Stigler (1964): “The detection of secret price cutting will of course be as difficult as interested people can make it.”

Signals on cheating, however, do exist. Although an unreasonably low price may reflect an abrupt decline in demand, if demand has a stochastic element, in our framework, which is not stochastic, cheating can easily be detected. Moreover, in continuous-time differential games, no momentary gains based on delayed actions of rivals exist, since reaction is instantaneous. The main source of potential instability in our game therefore arises from the fact that capital is a source of power for the bargaining agreements.

Our concern is with the existence of an equilibrium solution for which the collusive agreement remains in effect throughout the planning horizon. Such an equilibrium will be called stable.

Let  $\Omega_c = \prod_{i \in N} \Omega_{ci}$ . Let  $w \in \Omega_c$  be a strategy profile and let

$$\bar{w}_i = (w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n).$$

A *Nash equilibrium* for the collusive game (game  $G_c(K_0)$ ) is a strategy profile  $w^* \in \Omega_c$  such that  $w_i^* = (T_i^*, I_i^*(t), B(t))$  maximizes  $J_{ci}$  subject to (1), given  $\bar{w}_i^*$ . A Nash equilibrium for the collusive game is a *stable collusive equilibrium* if there does not exist an  $i$  such that  $T_i^*$  is finite. Note that even with infinite  $T_i^*$  the firm still plans two paths: one for the collusive agreement, as long as it holds, and one for the rivalry case, which the firm can use as a contingency strategy.

In the following proposition we show that the firm does not find it optimal to be the first to break the collusive agreement, given the strategies of its rivals. The economic intuition behind the result is derived from the proof and discussed below.

*Proposition 1.* Let  $\hat{T}(\bar{w}_i)$  be the minimum of  $T_j, j \in N - \{i\}$ . For a given  $\bar{w}_i$ , any strategy  $w_i$  for which  $T_i < \hat{T}(\bar{w}_i)$  is not optimal.

See the Appendix for the proof.

The main force behind this proposition is that the costate variables, or multipliers, are continuous at  $T_i$ . This forces the investment strategy to be continuous, and thus it allows us to compare the Hamiltonians at time  $T_i$ . Intuitively, they are continuous because the level of investment is directly related to the multipliers by the equation  $\partial H_i / \partial I_i = 0$ . Suppose the multipliers were not continuous at  $T_i$ . Just before  $T_i$  the firm invested according to  $\lambda_i(T_i^-)$ , but it already knew that at  $T_i$  the multiplier would make a discontinuous jump to  $\mu_i(T_i^+)$ , which implies that the investment made at  $T_i^-$  was not optimal. As the firm chooses  $T_i$  optimally, this excludes the possibility of a discontinuity.

*Theorem 2.* The collusive game  $G_c(K_0)$  satisfies the condition that for every initial capital stock  $K_0$ , there exists a stable collusive Nash equilibrium.

*Proof.* Consider  $I^*(t)$ , the Nash equilibrium of game  $B$  whose existence is assured by Theorem 1. Using  $I^*(t)$ , we construct a strategy profile  $w^*$  for game  $C$  such that

$$w^* = \{ \infty, I(t \times B) \} = \begin{cases} I^*(t) & \text{for } B(t) = 1 \\ I_r(t) & \text{for } B(t) = 0 \end{cases},$$

where  $I_r(t)$  can be a piecewise continuous function, such as the solution for game  $A$ . We have to show that  $w^*$  is a Nash equilibrium. Collusive stability will follow by definition of  $w^*$ . The best response for  $\bar{w}_i^*$  is  $w_i^*$ , since, from Proposition 1, if  $T_j^* = \infty$  for all  $j \in N - \{i\}$ , the optimal  $T_i$  is  $T_i^* = \infty$ . Now, given that  $T_j^* = \infty$  for all  $j \in N$ , since  $I^*(t)$  is a Nash equilibrium for game  $B$ ,  $I_i^*(t)$  is the best response of firm  $i$  against  $I_j^*(t)$  for  $j \in N - \{i\}$ . *Q.E.D.*

From the above arguments it is evident that every Nash equilibrium of game  $B$  induces a Nash equilibrium for game  $C$  that is stable. Still, games  $B$  and  $C$  are not equivalent, since the strategy space of game  $C$  allows for contingency strategies, and in addition the set of equilibrium points of game  $C$  is larger because it contains equilibria in which the collusion breaks down at some finite time. Using Theorem 1, we have the following corollary:

*Corollary.* If  $|\partial^2 \pi_{ri} / \partial K_i^2 + \partial^2 \Phi_i / \partial K_i^2| > \sum_{j \neq i} |\partial^2 \pi_{ri} / \partial K_i \partial K_j + \partial^2 \Phi_i / \partial K_i \partial K_j|$ , then there exists a unique stationary stable Nash equilibrium point, and from every initial capital stock  $K_0$  every stable Nash equilibrium solution converges to the unique stable stationary equilibrium point.

Theorem 2 guarantees only the *existence* of a stable equilibrium. Other equilibria, however, which are not collusive, exist as well.

### 6. Globally absorbing equilibrium

■ In this section we show that if we restrict the strategy space of each player such that he could without loss of generality limit the search to this set, then the resulting equilibrium is the stable collusive one. To show this in a formal manner we need some notions that are related to the ones developed by Kalai and Samet (1984).

*Definition 3.* A set  $W \subset \Omega_{ci}$  is *globally absorbing* if for every strategy  $\hat{w}_i \in \Omega_{ci} - W$  and  $\bar{w}_i = (w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n) \in \bar{\Omega}_{ci}$  there exists  $\tilde{w}_i \in W$  such that  $J_{ci}(\tilde{w}_i, \bar{w}_i) \geq J_{ci}(\hat{w}_i, \bar{w}_i)$ .

Intuitively, a set of strategies is globally absorbing if the player could without loss of generality limit his search to that set. More precisely, a set of strategies is globally absorbing if for every strategy not in the set and every conjecture about the competitors' strategies there is a still better (or at least not worse) strategy in the set. Thus, the sets of interest are the globally absorbing ones, since the players do not have any incentives to use strategies outside these sets.

*Definition 4.* A set  $\Lambda_i(T)$  is a *time retract* if it is the set of strategies  $w_i \in \Omega_{ci}$  for which  $T_i > T$ .

*Definition 5.* A *minimally globally absorbing time retract* is a time retract that is globally absorbing, and does not properly contain any globally absorbing time retract.

*Definition 6.* A strategy profile  $w^* = (w_1^*, \dots, w_n^*)$  is a *globally absorbing equilibrium* if  $w^*$  is a Nash equilibrium and every  $w_i^*$  belongs to a minimally globally absorbing time retract.

Intuitively, for any  $T$  a time retract is the set of strategies in which the firm waits longer than  $T$  before breaking the collusive agreement. If the firm does not lose by limiting its search for the best response to those strategies with the time for breaking the agreement longer than  $T$ , then such a time retract is globally absorbing. If, in addition, the time  $T$  for breaking the agreement is maximal, such a time retract is minimally globally absorbing.

The aim of the next theorem is to show that if we retract the strategy set of each player in a maximal manner, then the only resulting equilibrium is the stable collusive one, that is, the one in which the firms collude throughout the planning horizon.

*Theorem 3.* A Nash equilibrium of the game  $G_c(K_0)$  is stable collusive if and only if it is globally absorbing.

We need the following lemma to prove the theorem.

*Lemma.* Consider the following set:

$$\Lambda_i = \{w_i = (T_i, I_i(t, B)) \in \Omega_{ci} | T_i = \infty\}.$$

The set  $\Lambda_i$  is globally absorbing.

*Proof.* The proof will be carried out in two steps: for every  $\bar{w}_i$  let  $BR(\bar{w}_i) \subset \Omega_{ci}$  be the set of all best response strategies of firm  $i$  against  $\bar{w}_i$ .

(i)  $BR(\bar{w}_i) \cap \Lambda_i \neq \emptyset$ . First, observe that  $BR(\bar{w}_i)$  is not empty. This is so since for every  $\bar{w}_i$ , from Proposition 1, the best response of firm  $i$  is not to be the first to break the agreement. Now that the time  $T$  for breaking the agreement is given, standard existence theory for optimal control assures us of the existence of a best response  $w_i$  (see, for example, Baum (1976)). Observe further that if  $\hat{w}_i = (\hat{T}_i, \hat{I}_i(t, B)) \in BR(\bar{w}_i)$ , then if  $\hat{w}_i \notin \Lambda_i$ , it follows that  $\hat{T}_i < \infty$ . From Proposition 1 we know that  $\hat{T}_i \geq \hat{T}_i(\bar{w}_i)$ , where  $\hat{T}_i(\bar{w}_i)$  is the minimum of  $T_j, j \in N - \{i\}$ . Define  $w_i^\infty = (\infty, \tilde{I}_i(t, B))$ . Since  $\hat{w}_i$  and  $w_i^\infty$  share the same  $\tilde{I}_i(t, B)$  and the time for breaking the agreement continues to be  $\hat{T}_i(\bar{w}_i)$ , it follows that  $J_{ci}(\hat{w}_i, \bar{w}_i) = J_{ci}(w_i^\infty, \bar{w}_i)$ . Therefore,  $w_i^\infty \in BR(\bar{w}_i)$ . By definition  $w_i^\infty \in \Lambda_i$ .

(ii) For every  $\tilde{w}_i \in \Omega_{ci} - \Lambda_i$  and every  $\bar{w}_i$ , if  $\tilde{w}_i \notin BR(\bar{w}_i)$ , then by (i) there exists  $\hat{w}_i \in \Lambda_i \cap BR(\bar{w}_i)$  such that  $J_{ci}(\hat{w}_i, \bar{w}_i) > J_{ci}(\tilde{w}_i, \bar{w}_i)$ . If  $\tilde{w}_i \in BR(\bar{w}_i)$ , then by (i) there exists  $\hat{w}_i \in \Lambda_i \cap BR(\bar{w}_i)$  such that  $J_{ci}(\hat{w}_i, \bar{w}_i) = J_{ci}(\tilde{w}_i, \bar{w}_i)$ . Thus, in both cases we have found a best response strategy with respect to  $\bar{w}_i$  that belongs to  $\Lambda_i$ .

*Proof of Theorem 3.* Using the lemma just proved, we conclude that for every  $T, \Lambda_i(T)$  is a globally absorbing time retract since it contains  $\Lambda_i$ . Moreover, it is clear that the only minimal globally absorbing time retract is  $\Lambda_i$ . Given a stably collusive Nash equilibrium  $w^*$  whose existence is assured by Theorem (2),  $w_i^* \in \Lambda_i$  for every  $i$ , and thus  $w^*$  is a globally absorbing equilibrium. Given a nonstable collusive Nash equilibrium  $v^*$ , there exists at least one firm  $j$  such that  $T_j^*$  is finite. Therefore,  $v_j^* \in \Lambda_j(T_j^*)$ , which is not minimal. *Q.E.D.*

The refinement of equilibrium that we use here in effect limits the firm to search for the best response by asking how long it should wait before being the first to break the collusive agreement. Waiting longer limits the search to a smaller set. The firm, however, does not lose by waiting longer, and concludes that it should never be the first to break the agreement on prices. Note that this restriction of strategies is self-enforcing, since no firm has any incentive to choose strategies outside its retract.

In equilibrium all firms will have the same consideration, so the only equilibrium is

the collusive one.<sup>4</sup> From Theorem 3 we can conclude that long-run competition is not likely to have a destabilizing effect on short-run collusion. In other words, collusion on prices can survive competition in investment. The basic force behind this result is the fact that capital cannot be instantaneously changed. Moreover, large immediate changes are prohibitively costly. Formally, this results from the strict convexity of the cost function plus the continuity requirements we imposed on the model. First, the convexity assumption is the one that makes capital and investment in capital long-term variables. The difference is apparent when one considers the two classical capital accumulation problems of a single monopolist by Nerlove and Arrow (1962) and by Gould (1973). In the first capital is instantaneously changed to its optimal level by an infinite injection of investment. This is optimal since the cost of investment is linear. In the latter the firm gradually changes the level of capital since instantaneous changes are prohibitively costly. The optimal level that is reached instantaneously in the first model is not reached at all at a finite time in the second. This forces continuity in the latter model and in ours. In particular, it forces the investment path to be continuous, even when the firm decides to break the collusive agreement.

Given the complete information structure and the continuous time framework of the model, if collusive price setting were not stable, firms would know that, and would expect collusion to break down at some particular date  $T$ . But, given the noncontingent nature of the investments in capital stocks, firms would adjust their investments and stocks towards a suitable level for price competition by time  $T$ . Because of the strict convexity of the cost function, firms make the adjustment such that both stocks *and* investment are continuous at time  $T$ .

Intuitively, investments are continuous at time  $T$ , since first-order conditions for maximization imply that they are proportional to the multipliers (auxiliary variables). Suppose that the multipliers were not continuous at  $T$ . Just before  $T$  the firm invested according to the multiplier at that time, but it already knew that at  $T$  the multiplier would make a discontinuous jump, which implies that the investment made just before  $T$  was not optimal.

Part of the definition of collusive pricing is that, given any fixed capital stocks, *instantaneous* profits are higher than with competitive pricing.

Now note that the multiplier measures the addition to aggregated discounted profits due to an additional unit of capital. This means that because of the above continuity, *aggregate discounted profits* will be higher under the collusive arrangement, even when we account for the possibly higher cost of investment necessary to maintain the capital to support this arrangement.

## 7. The possibility of overcapitalization

■ In the preceding analysis we have shown the existence of two distinct paths that might follow from the game: one on which the players collude and one on which they engage in rivalrous behavior. It is of interest to determine on which of the two paths the overall capital

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<sup>4</sup> This notion of globally absorbing equilibrium is closely related, but not equivalent to, the persistent equilibrium notion of Kalai and Samet. There are two main differences. First, our absorption is a global property and is not defined in a neighborhood of a retract. This allows us to restrict the strategy space of each player *unilaterally*, and his agreement to this retraction does not depend on the retract of other players. In a locally absorbing retract all strategy spaces are retracted simultaneously, and the agreement to this retraction depends on the fact that the strategy space of the rest of the players is retractable as well. Second, we are dealing with a specific form of retract, namely, time retracts. The reason we cannot use the Kalai and Samet (1984) definitions and therefore their result about the existence of a persistent equilibrium is that the strategy space of each individual player is not restricted in our case to be compact. This, of course, makes our existence theorem more difficult to prove in addition to preventing us from using the persistence result, but there is no *a priori* reason to restrict the strategy space in our game to a more restrictive space than  $\Omega_{ci}$ , which is not compact.

invested in the industry is larger. This issue has been given considerable attention when the capital in question is goodwill and the investment is advertising. From Theorem 2 and its corollary we are assured not only of the existence of an equilibrium, but also of its convergence to a unique stationary point, regardless of the initial conditions. Thus, we can investigate the behavior of the market at the steady state. Continuity will guarantee that the same behavior in terms of over- or undercapitalization will carry over the neighborhood of the stationary point. For simplicity, we deal with the duopoly case only.

*Theorem 4.* Let game  $G_c(K_0)$  satisfy the assumption that guarantees global asymptotic stability, i.e.,

$$|\partial^2 \pi_i / \partial K_i^2| > |\partial^2 \pi_i / \partial K_i \partial K_j| \quad \text{for } i \neq j.$$

Let  $K_{ci}^*$  and  $K_{ri}^*$  denote the level of capital firm  $i$  achieves at the unique stationary point at collusion and at rivalry, respectively. Then

$$\partial \Phi_i / \partial K_i > 0 \quad \text{if and only if} \quad \sum_i K_{ci}^* > \sum_i K_{ri}^*.$$

*Proof.* Using the Appendix for the definition of the current value Hamiltonian and the necessary conditions, we find that at the stationary equilibrium for game  $B$  the following holds:

$$(r + \delta_i)C'_i(\delta_i K_{ci}^*) = \pi_{ri}^i(K_c^*) + \Phi_i^i(K_c^*). \tag{7}$$

Similarly, for game  $A$  we have

$$(r + \delta_i)C'_i(\delta_i K_{ri}^*) = \pi_{ri}^i(K_r^*). \tag{8}$$

We shall prove that  $\partial \Phi_i / \partial K_i > 0$  implies  $\sum_i K_{ci}^* > \sum_i K_{ri}^*$ . *Mutatis mutandis*, we can use the same proof to prove the reverse implication. Let  $\pi_{ri}^{ij} = \partial \pi_{ri} / \partial K_i \partial K_j < 0$ . The case in which the reverse condition holds can be similarly treated.

First, assume to the contrary that  $K_{ri}^* \geq K_{ci}^*$  for  $i = 1, 2$ . We now claim that

$$\pi_{r1}^1(K_r^*) \geq \pi_{r1}^1(K_c^*) + \Phi_1^1(K_c^*) > \pi_{r1}^1(K_c^*). \tag{9}$$

The first inequality follows from the facts that  $K_{r1} > K_{c1}$  and  $C''_i > 0$  and from equations (7) and (8). The second inequality follows from the assumption that  $\Phi_i^i > 0$ .

Using the mean value theorem, we find that for some mean value of  $\tilde{K}$  the following is true:

$$\pi_{r1}^1(K_r^*) - \pi_{r1}^1(K_c^*) = (K_{r1}^* - K_{c1}^*)\pi_{r1}^{11}(\tilde{K}) + (K_{r2}^* - K_{c2}^*)\pi_{r1}^{12}(\tilde{K}). \tag{10}$$

Observe that the negativity of  $\pi_{r1}^{11}$  and  $\pi_{r1}^{12}$  and the fact that  $K_{r1}^* > K_{c1}^*$  imply that the right-hand side of equation (10) is nonpositive, which contradicts inequality (9).

Thus, it is not true that  $K_{ri}^* \geq K_{ci}^*$  for  $i = 1, 2$ , and therefore either  $K_{ri}^* < K_{ci}^*$  for  $i = 1, 2$ , so  $\sum_i K_{ri}^* < \sum_i K_{ci}^*$  or, without loss of generality,  $K_{r1}^* \geq K_{c1}^*$ , but  $K_{r2}^* \leq K_{c2}^*$ . From

inequality (9) it follows that the left-hand side and therefore the right-hand side of equation (10) are strictly positive. By the assumption that  $|\pi_{r1}^{11}(\tilde{K})| > |\pi_{r1}^{12}(\tilde{K})|$ , we have  $K_{c2}^* - K_{r2}^* > K_{r1}^* - K_{c1}^*$ , and so  $\sum_i K_{ci}^* > \sum_i K_{ri}^*$ . *Q.E.D.*

The theorem above states that industry overcapitalization depends on the sign of  $\partial \Phi_i / \partial K_i$ . If each firm's benefits from engaging in collusive behavior are increasing in capital, i.e.,  $\partial \Phi_i / \partial K_i > 0$ , we can expect overcapitalization to occur.

This issue of overcapitalization is also related to the case of collusion with capacity constraints discussed by Brock and Scheinkman (1985). Their finding is that in a repeated game where tacit collusion is present, excessive entry may occur. In such an industry, where the capacity of each firm is given, overcapitalization is a direct result. Note that in addition

to the difference in the game-theoretic structure, the articles differ in the treatment of capital, where a generic property in our setting is the ability of the firm to change its capital level.

### 8. Conclusion

■ While discussing strategic interaction in oligopolistic markets, one must be aware of the distinction between variables, such as capital and capacity, that are “sticky” in the short run but can be changed in the long run and variables that can be changed instantaneously, such as prices.

In this article we have presented a model in which firms compete in a long-run variable and collude in the short run. Our main concern is whether the long-run competition in investment may destabilize the short-run collusion in prices.

This destabilizing effect can occur when the collusion in prices makes long-run competition so costly that it yields lower profits than in the case in which the firms compete in both the short run and the long run. We have shown that long-run competition does not have a destabilizing effect on short-run collusion. In addition, we have shown the conditions under which overcapitalization occurs.

### Appendix

■ **Proof of Proposition 1.** We must show that for a given  $\bar{w}_i$  any strategy for which  $T_i < \hat{T}(\bar{w}_i)$  is not optimal. Assume to the contrary that there exists an optimal time  $T_i < \hat{T}(\bar{w}_i)$ . The method of proof for this case will follow a variation on Amit (1986), which is an extension of a method by Kamien and Schwartz (1981). While making the decision to break the collusive agreement at time  $T_i$ , the firm realizes that, as a consequence, the paths of investment and capital stocks of its rivals will change accordingly. Thus, the firm chooses time  $T_i$  and investment path to maximize its total discounted profits as follows:

$$\int_0^{T_i} \{ \pi_{\pi}(K) + \Phi_i(K) - C_i(I_i) \} e^{-\pi t} dt + \int_{T_i}^{\infty} \{ \pi_{\pi}(K) - C_i(I_i) \} e^{-\pi t} dt,$$

subject to (1), and for  $j \neq i$

$$K_j(t) = \begin{cases} K_{cj}(t) & \text{for } 0 \leq t \leq T_i \\ K_{rj}(t) & \text{for } T_i \leq t, \end{cases}$$

where  $T_i \leq \hat{T}$  and  $K_i(T_i)$  is free.  $K_{cj}(t)$  is the capital path of firm  $j$  that is induced from its investment policy  $I_j(t, 1)$  under a collusive agreement, and  $K_{rj}(t)$  is the corresponding path when firm  $j$  is in a rivalry situation. Define the following two current value Hamiltonians  $H_1$  and  $H_2$  corresponding to the two different time periods as

$$H_{1t} = \pi_{\pi}(K) + \Phi_i(K) - C_i(I_i) + \lambda_i I_i - \lambda_i \delta_i K_i$$

$$H_{2t} = \pi_{\pi}(K) - C_i(I_i) + \mu_i I_i - \mu_i \delta_i K_i.$$

At time  $T_i$  it follows from Amit (1986) that  $\lambda_i(T_i) = \mu_i(T_i)$ , and that  $H_1(T_i) = H_2(T_i)$  if  $0 < T_i < \hat{T}$ ,  $H_1(T_i) < H_2(T_i)$  if  $0 = T_i$ , and  $H_1(T_i) > H_2(T_i)$  if  $T_i = \hat{T}$ .

The relevant necessary conditions for optimality are

$$\partial H_{1t} / \partial I_i = -C'_i + \lambda_i = 0$$

$$\partial H_{2t} / \partial I_i = -C'_i + \mu_i = 0.$$

The equality of  $\mu_i$  to  $\lambda_i$  at time  $T_i$ , therefore, implies the equality of  $I_i^+$  and  $I_i^-$  at time  $T_i$ . The continuity of the capital paths implies that  $K_{cj}(T_i) = K_{rj}(T_i)$ . The individual rationality condition implies that  $\Phi$  is positive, and thus at time  $T_i$  we have that  $H_1(T_i) > H_2(T_i)$ . Thus, there does not exist such a finite  $T_i^* < \hat{T}$ , and we have our desired contradiction. *Q.E.D.*

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