

INNOVATION DIFFUSION IN THE PRESENCE OF SUPPLY RESTRICTIONS

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Innovation diffusion models are developed to represent the spread of a new product from its manufacturer(s) to its ultimate users. In the marketplace, however, the growth of a new product can be retarded by supply restrictions such as the unavailability of the product due to limitations on the production capacity or difficulties encountered in setting up distribution systems. In the presence of supply restrictions, diffusion models must be developed to capture the dynamics of supply restrictions and to allow management to evaluate the impact of such supply restrictions on the growth of a new product in the market place. Based on the Bass innovation diffusion model, the objective of this paper is to suggest a parsimonious diffusion model that integrates the demand side dynamics with the supply side restrictions. The sensitivity of innovation diffusion patterns in the presence of supply restrictions is examined. An application examining the diffusion of new telephones in Israel is documented to illustrate the usefulness of the proposed model.

(New Products; Forecasting)

1. Introduction

Many studies in the areas of marketing, technological forecasting and economics have attempted to model the time-dependent aspects of the innovation diffusion process, the process by which an innovation is communicated through certain channels over time among the members of a social system (Rogers 1983). The underlying behavioral theory in the development of these models is that the innovation is first adopted by innovators who, in turn, influence others (via word-of-mouth) to adopt it. The classical Bass (1969) model in marketing describes the diffusion process by the following differential equation:

$$\frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m} N(t)[m - N(t)], \quad (1)$$

where $N(t)$ is the cumulative number of adopters at time t , m is the total population of potential adopters, p is the coefficient of innovation and q is the coefficient of imitation. The first term in equation (1) denotes the adoptions by innovators and the second term adoptions by imitators.

In recent years, a number of efforts have been made in the marketing science literature to incorporate the effect of marketing mix variables, especially price (Bass 1980, Jain and Rao 1990), advertising (Horsky and Simon 1983) and distribution (Jones and Mason 1989) into this model (for a review of such efforts, see Kalish and Sen 1986, and Mahajan,

Muller and Bass 1990). While these contributions provide valuable insights into the descriptive and the normative influences of marketing mix variables on the growth pattern of a new product, there is a conceptual limitation associated with these models. These modeling efforts ignore the impact of supply restrictions, such as the unavailability of the product due to limitations on the production capacity or difficulties encountered in setting up distribution systems, on the diffusion process. Although well documented in the diffusion literature (Brown 1981), this limitation was first identified in the marketing science literature by Simon and Sebastian (1987) in their study related to the effect of advertising on the diffusion of new telephones in West Germany. In fact, as articulated by Simon and Sebastian (1987, p. 460): “It is important to realize that the Bass-diffusion model is solely concerned with the demand side. An empirically observed diffusion pattern may, however, be governed by bottlenecks on the supply side (production capacity, distribution, etc.), so that the “natural” demand process is decelerated or retarded . . .” In the presence of supply restrictions diffusion models must be developed to (a) capture the dynamics of supply restrictions, and (b) allow management to evaluate the impact of such supply restrictions on the growth of a new product in the marketplace.¹

Based on the Bass diffusion model, we suggest a parsimonious model that incorporates supply side restrictions.

2. Modeling the Impact of Supply Restrictions in the Bass-Diffusion Model

Figure 1 presents our conceptualization of the diffusion process under supply restrictions. The figure depicts the customers flow in the presence of supply restrictions. The Bass model can be thought of as a two-stage diffusion model, i.e., it formulates the customers flow from Potential Adopters to Adopters. In the presence of supply restrictions,

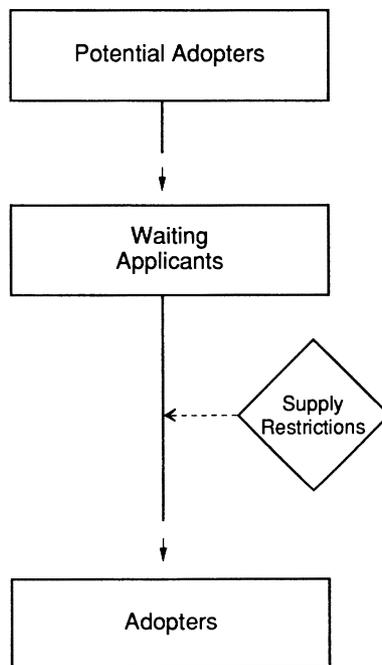


FIGURE 1. Flow Diagram with Supply Restrictions.

¹ The influence of supply restrictions on consumer preferences and the demand of a new product has been modeled in a multiple-equation framework by Urban, Hauser and Roberts (1990). They study the influence of production constraints pertaining to automobiles within a model that includes advertising, dealer visits, and word-of-mouth.

not every customer who requests the product receives it at the time of the request (e.g., applicants for new telephones in West Germany). There is a pool of customers, termed *Waiting Applicants* in Figure 1, that await the availability of the product. That is, the diffusion model becomes a three-stage model with the customer flow from Potential Adopters to Waiting Applicants and then from Waiting Applicants to Adopters. Of course, when there are no supply restrictions, the number of Waiting Applicants is zero since every customer who requests for the product receives it at the time of request.

In order to represent mathematically the customer flow among the three stages, let $A(t)$ = number of Waiting Applicants at time t . Because Waiting Applicants have already expressed a commitment to adopt the product, the remaining potential applicants, $(m - A(t) - N(t))$, can be influenced (via word-of-mouth) to request for the product and hence generate demand for it. If we assume that both Waiting Applicants and Adopters generate this influence, following the Bass model, the customers flow across the three stages can be expressed by the following state equations:

$$\frac{dA(t)}{dt} = \left(p + \frac{q_1}{m} A(t) + \frac{q_2}{m} N(t) \right) (m - A(t) - N(t)) - c(t)A(t), \tag{2}$$

$$\frac{dN(t)}{dt} = c(t)A(t). \tag{3}$$

As expressed in equation (2), the rate of change of Waiting Applicants is increased by *New Applicants* (the first term in equation (2)) generated by the influence of Waiting Applicants and Adopters on potential applicants, reflected by the coefficients of imitation q_1 and q_2 , respectively. It is, however, decreased by the conversion rate of Waiting Applicants to Adopters, moderated by the supply coefficient $c(t)$. Equation (3) represents this conversion where the supply coefficient $c(t)$ captures the supply restrictions at time t . It should be noted that like the Bass model the coefficients p and q_2 are nonnegative; q_1 on the other hand can be positive or negative depending upon the nature of the word-of-mouth communicated by the Waiting Applicants.

3. Nature of Supply Restricted Diffusion Patterns

Simon and Sebastian (1987) suggest that supply restrictions generate diffusion patterns that are negatively skewed, i.e., slow growth, fast decline. We now consider the implications of equations (2) and (3) for the growth patterns of New Applicants, Waiting Applicants and Adopters over time.

3.1. Growth Patterns for New Applicants

To capture the dynamics of New Applicants, let $Z(t) = A(t) + N(t)$ denote the sum of Waiting Applicants and the cumulative number of Adopters at time t . The evolution of $Z(t)$ over time can be depicted by adding equations (2) and (3). That is,

$$\frac{dZ(t)}{dt} = \frac{dA(t)}{dt} + \frac{dN(t)}{dt} = \left(p + \frac{q_1}{m} A(t) + \frac{q_2}{m} N(t) \right) (m - A(t) - N(t)). \tag{4}$$

Equation (4) includes the first term from equation (2) and captures the flow of customers from Potential Adopters to Waiting Applicants. It represents the rate of New Applicants who request for the product. Note that this rate, although not directly affected by the nature of supply restrictions, is indirectly affected by the restrictions because they influence the size of the groups $A(t)$ and $N(t)$ as indicated in equation (3).²

² If we assume that $q_1 = q_2 = q$ (i.e., both Waiting Applicants and Adopters generate the same intensity of influence on potential applicants), equation (4) degenerates into the Bass model.

3.2. Growth Patterns for Waiting Applicants and Adopters

As depicted in Figure 1, the customers flow from Waiting Applicants to Adopters is controlled by supply restrictions. Such restrictions, therefore, do influence the growth patterns for Waiting Applicants and Adopters. In this article, we consider supply restrictions that represent situations where the number of product units made available at any time t bears a relationship with $A(t)$, the size of Waiting Applicants (e.g., in certain service organizations where the product availability can be adjusted by managing the workforce). As suggested by equation (3), in these situations, the supply coefficient $c(t)$ determines the relationship between Adopters and Waiting Applicants:

$$c(t) = \frac{dN(t)/dt}{A(t)} = \frac{n(t)}{A(t)}. \quad (5)$$

That is, the supply coefficient $c(t)$, controls the flow of customers from Waiting Applicants to Adopters and hence influences the growth pattern of Adopters. Using equation (5), one can determine the nature of supply restrictions over time. Unfortunately, however, it is not possible to solve equations (2) and (3) to obtain a closed form solution for $N(t)$.

If we assume that $c(t) = c$, a constant, then equation (3) which represents the relationship between Adopters and Waiting Applicants can be written as

$$dN(t)/dt = cA(t). \quad (6)$$

In this case, $1/c$ may be interpreted as the expected time a Waiting Applicant who has decided to adopt a product has to wait before he receives the product.³ In the data on telephone diffusion in Israel analyzed in §4 of this paper, we find that $c = 0.273$ which implies that on the average a Waiting Applicant had to wait 3.7 years before he received the telephone. Simon and Sebastian (1987, p. 460) report that in Germany “in some regions the waiting time for a new telephone ran up to two years,” implying that $c = 0.5$.

3.3. Negatively Skewed Growth Patterns

In order to examine the impact of supply restrictions on the diffusion patterns, we generated diffusion patterns by numerically solving the discrete analog of the supply-restricted diffusion model, equations (2) and (3), and using the initial conditions $A(0) = N(0) = 0$. In generating these different patterns, ten different values of c , from 0.1 to 1.0 in increments of 0.1 were used (note that $c = 1$ generates the Bass model). The values of the diffusion parameters were fixed at $p = 0.017$, $q_2 = 0.335$ and $m = 54$ million.⁴

Figure 2 plots these patterns for five different values of c ($c = 0.05, 0.1, 0.2, 0.4$, and 1) and three different values for q_1 ($q_1 = .2, 0, -0.2$). It can be noted in Figure 2 that, consistent with the observation made by Simon and Sebastian (1987), supply restrictions tend to generate diffusion patterns that are negatively skewed. As q_1 decreases, diffusion patterns tend to be more negatively skewed.

³ Suppose at time 0 everyone in the population is a Waiting Applicant; then $A(0) = m$ and equation (2) reduces to $dA/dt = -cA(t)$. Solving this differential equation, we get $A(t) = m \exp[-ct]$. Substituting this in equation (3) and dividing both sides by m we get $f(t) = c \exp[-ct]$ where $f(t) = (1/m)(dN(t)/dt)$ is the probability that a Waiting Applicant becomes an Adopter in time $(t, t + dt)$. Clearly, the mean of this probability distribution is $1/c$.

⁴ The values of p and q_2 are close to the average values of these parameters for the eleven consumer durables analyzed by Bass (1969). The value of m was taken as the number of US households in 1961 since the data on most of these products were up to 1961 (Schmittlein and Mahajan 1982).

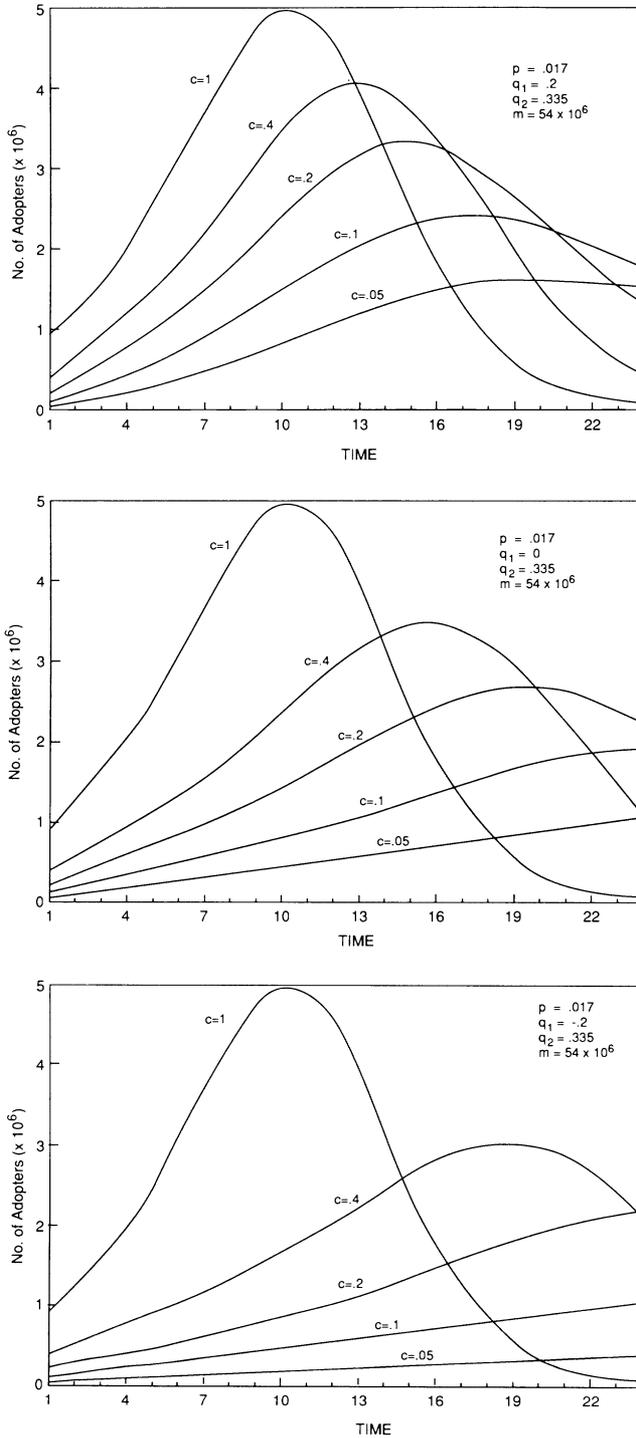


FIGURE 2. The Influence of Supply Restrictions on Diffusion Patterns.

4. Diffusion of New Telephones in Israel

In documenting the diffusion of a product in the presence of supply restrictions, Simon and Sebastian (1987) studied the diffusion of new telephones in West Germany. Due to the proprietary nature of their application, these data understandably were not made

available for this paper (private communication, Sebastian 1987). Therefore, in order to illustrate the application of the proposed supply restricted diffusion model, we report here its application to the diffusion of new telephones in Israel.

Figure 3 depicts the number of New Applicants, the number of Waiting Applicants and the number of telephones installed in Israel from 1949–1987. These data were collected from the Annual Statistical Yearbook of BEZEK, the Israeli Communication Corporation (ICC). Like the diffusion of new telephones in West Germany reported by Simon and Sebastian (1987), the Israeli data have the following distinguishing characteristics. First, as indicated in Figure 3, since 1949 the number of New Applicants for telephones has exceeded the supply creating a consistent pool of Waiting Applicants. For the time period 1969–79, the number of installations seem to have remained constant around 44,000. Since 1979, however, attempts seemed to have been made to increase the supply to its present level of 105,000 installations per year. This increase seems to have caused a downward trend in the number of waiting applicants (see Figure 3(b)). Second, the supply is under the control of the ICC. There is no competition and the product itself has not changed over time. Figure 3(c) represents the supply distribution controlled by supply restrictions.

Given the nature of the data, the following questions can be raised now:

(1) Can the growth of New Applicants for telephones in Israel be described by the proposed model, equations (2) and (3)?

(2) If ICC maintains the present level of installations which is 105,000 per year, can we predict the number of customers that will wait each year to receive their installations? By what year will there be no Waiting Applicants?

In order to examine these questions, the data on the diffusion of new telephones in Israel were analyzed by using the results derived in §3. The parameters of the model describing the growth of New Applicants over time, equation (4), were estimated by using the nonlinear least squares procedure. For the data describing the number of New Applicants from 1948 to 1987, the following parameter estimates (standard errors) and fit statistics were obtained: $\hat{p} = 0.0027$ (0.0015), $\hat{q}_1 = -0.3077$ (0.1315), $\hat{q}_2 = 0.2551$ (0.0334) and m (thousands) = 1747.29 (71.99), $R^2 = 0.887$. Note that \hat{q}_1 is negative. Thus, as the number of Waiting Applicants increases, they tend to communicate negative information about the product influencing the rate of new potential applicants.

To obtain an estimate for the supply coefficient, we used equation (6) and obtained $\hat{c} = 0.273$ (0.015) and $R^2 = 0.903$. This implies that using a constant supply coefficient, we are able to explain 90.3% of the variation in the relationship between the rate of adoption and the size of the group of Waiting Applicants. As pointed out earlier, this value of \hat{c} implies that the expected waiting time to receive a telephone connection is 3.7 years.

To answer the second question raised above, it is necessary to predict the number of Waiting Applicants, $A(t)$, at any time t . Since the adopter distribution, $N(t)$, is known (controlled by the number of installations made available by ICC; see Figure 3c), and the number of New Applicants can now be predicted by using equation (4), the number of Waiting Applicants can be predicted by noting that $A(t) = Z(t) - N(t)$. Using this equation, we find that the predicted values of $Z(t)$ for 1988 and 1989 are 1.46 and 1.518 million respectively. The cumulative number of lines (i.e., $N(t)$) connected by the end of 1988 and 1989 should be 1.418 and 1.523 million, respectively if ICC maintained the present level of installations (i.e., 105,000 per year). These results suggest that, if the present level of installation was maintained, the ICC will have no Waiting Applicants at the end of 1989. At the end of 1988, however, there should have been 42,000 Waiting Applicants. These forecasts are based on the fact that $\hat{m} = 1.75$ millions. If however the market potential changes (e.g. due to population growth or new applications of telephone lines such as fax) then we need to use an external estimate of m and recalibrate the model (Heeler and Hustad 1980).

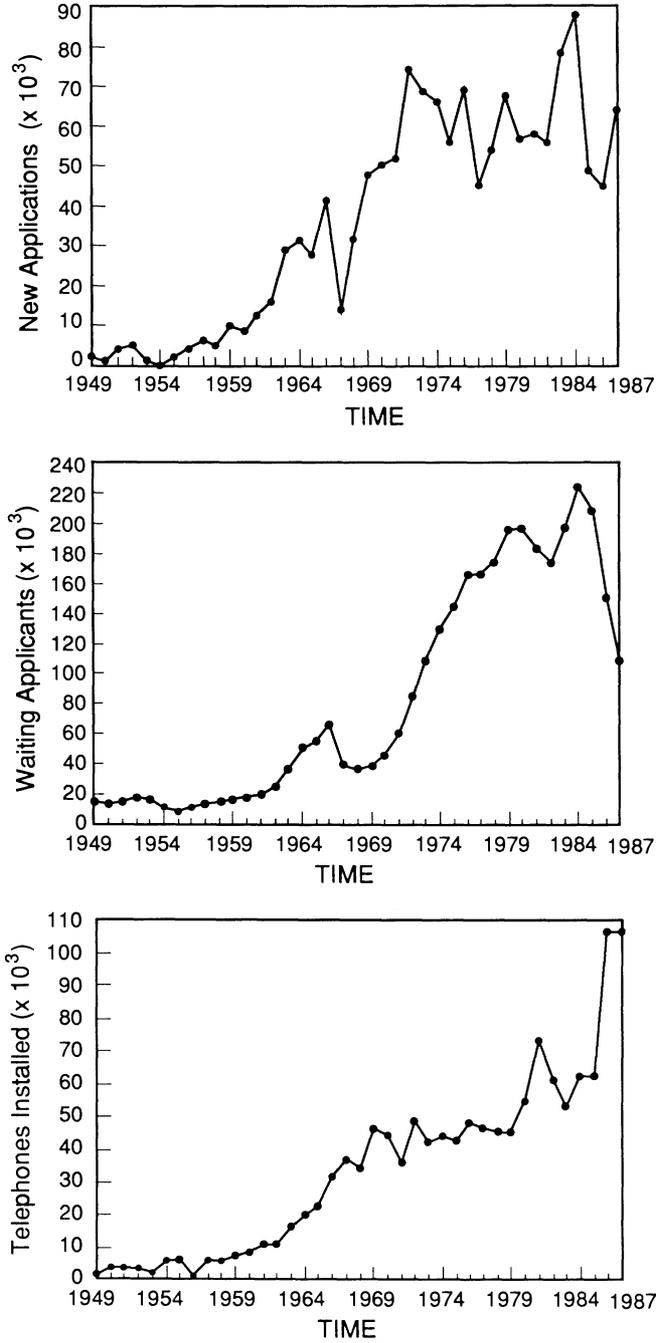


FIGURE 3. Diffusion of New Telephones in Israel. (1949–1987) (a) Number of New Applicants per Year. (b) Number of Waiting Applicants per Year. (c) Number of Telephones Installed per Year.

5. Summary

The goal of the present paper was to investigate the influence of supply restrictions on the diffusion of innovations. Toward this goal, the paper achieves the following:

(a) It extends the Bass-diffusion model to incorporate supply restrictions. [For other formulations, see recent studies by Simon and Sebastian (1987) and Urban, Hauser and Roberts (1990).]

(b) It demonstrates that supply restrictions (as modeled) can generate diffusion patterns that are negatively skewed.

(c) It applies the supply restricted diffusion model to forecast the demand of new telephones in Israel. Given the present supply policy of the Israeli Communication Corporation we can predict demand for telephones and the number of Waiting Applicants in further coming years.

The paper, thus, makes a contribution to the emerging diffusion modeling literature by suggesting a parsimonious formulation that integrates both the supply and the demand sides of the diffusion process. However, several issues need further investigation. These include (a) understanding the role of advertising and determining the optimal advertising strategy in the presence of supply restrictions, (b) determining the optimal distribution policy assuming that supply of the product can be controlled by management, (c) analyzing the influence of the negative word-of-mouth due to Waiting Applicants on the generation of New Applicants for the product, and (d) evaluating diffusion patterns under various scenarios for the size of Waiting Applicants (e.g., maximum number of Waiting Applicants allowed at any time). In that respect, the model may provide a mechanism to directly link diffusion dynamics with management capacity decisions related to product availability.⁵

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