Majority choice and the objective function of the firm under uncertainty: reply

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■ We are grateful to Professor Winter (1981) for pointing out a technical oversight in our paper (Benninga and Muller, 1979). Winter's main claim, which he states in his equations (2)–(5), is that we should have allowed all portfolio choices to vary when the consumer determines his choice for the optimal level of b_j (i.e., when computing our equation (14) to get (17)). This oversight does not invalidate our results, however, and it is not necessary to assume that current spanning holds in a neighborhood of the utility-maximizing production plan. To see this, write

$$f_{ih} = f_{ih}(b_1, \ldots, b_j, \ldots, b_J), \qquad h = 1, \ldots, J. \tag{1}$$

From (4) and (5) of our article, we have:

$$\frac{\partial x_{i0}}{\partial b_j} = \bar{f}_{ij}\left[\frac{\partial p_j}{\partial b_j} - p_b\right] - \sum_h \left[\frac{\partial f_{ih}}{\partial b_j}\right] p_h - f_{ij}\frac{\partial p_j}{\partial b_j} \tag{2}$$

$$\frac{\partial x_{im}}{\partial b_j} = \sum_h \frac{\partial f_{ih}}{\partial b_j} y_{hm} + f_{ij}\frac{\partial y_{jm}}{\partial b_j}. \tag{3}$$

Now $\partial U_i/\partial b_j = 0$ at b_j^*, i.e. (14) holds if

$$0 = \bar{f}_{ij}\left[\frac{\partial p_j}{\partial b_j} - p_b\right] + \sum_h \frac{\partial f_{ih}}{\partial b_j}\left[\sum_m q_m^i y_{hm} - p_h\right] + f_{ij}\left[\sum_m \frac{\partial y_{jm}}{\partial b_j} - \frac{\partial p_j}{\partial b_j}\right]. \tag{4}$$

The second term on the right-hand side of the last expression is zero by our equation (9). Evaluating f_{ij} at $(b_1^*, \ldots, b_j^*, \ldots, b_J^*)$, we get $f_{ij} = 0$ by current spanning. Thus (17) is established even though portfolio choices are assumed to depend on b_j, and this assumption does not alter our results. The assumption that current spanning holds only at b_j^* is thus sufficient to establish Theorem 1.

In his note Winter makes a number of other comments. The first of these is that current spanning implicitly assumes Ekern-Wilson (E-W) spanning (1974). In fact, however, if the last sentence of Winter's footnote 1 is true, Winter has shown that

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$$\text{not current spanning} \Rightarrow \text{not E-W spanning}.$$

This, of course, is equivalent to

$$\text{E-W spanning} \Rightarrow \text{current spanning},$$

which means that current spanning need not imply E-W spanning. In fact, as we showed in our paper by means of counterexamples, E-W spanning and current spanning are disjoint conditions (see Section 4 of our paper).

Another comment made by Winter is that the value of the firm (Winter's equation (1), and our equation (12)) depends only on market prices. Given utility-maximizing portfolios for given firm b_j's, this is true and may readily be deduced from our equation (9). What is not true, however, is that individuals will perceive changes in net firm value to be dependent only on market prices. In other words, the derivative of the present value of the firm will involve implicit prices, and it is these derivatives which determine our equilibrium.

Finally, Winter's last comment—that our results are limited to the one-dimensional case—is partially true. As we stated in Section 7, our treatment can be extended to the multicommodity case if we use the retained earnings approach. This means, as Winter correctly states, that firms do not "commit themselves to certain production processes and outputs before the resolution of all uncertainty." In this sense our model is indeed less general than the Arrow-Debreu framework.

References

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