easyJet® Pricing Strategy: Should Low-Fare Airlines Offer Last-Minute Deals?

Appendix

Proposition 1: The general optimization problem is represented by:

Period 2: $Max\ p_2\ (1-\gamma)N_B\ \theta\left(\overline{\alpha}-p_2\right)$

Subject to $\quad C-N_T\left(\alpha-p_1\right)-\gamma N_B\left(\overline{\alpha}-\alpha\right)-\theta(1-\gamma)N_B\left(\overline{\alpha}-p_2\right)>0$

Period 1: $Max\ p_1\ [N_T\ \left(\alpha-p_1\right)+\gamma N_B\left(\overline{\alpha}-\alpha\right)]+\prod_2$

Subject to $\quad C-N_T\left(\alpha-p_1\right)-\gamma N_B\left(\overline{\alpha}-\alpha\right)-\theta(1-\gamma)N_B\left(\overline{\alpha}-p_2\right)>0$ and $\alpha - p_1 > 0$

Thus, we can define Lagrangian in period 2 with coefficient λ and in period 1 with coefficients δ and μ.

$$L_2 = p_2\ (1-\gamma)N_B\ \theta\left(\overline{\alpha}-p_2\right)+\lambda[\,C-N_T\left(\alpha-p_1\right)-\gamma N_B\left(\overline{\alpha}-\alpha\right)-\theta(1-\gamma)N_B\left(\overline{\alpha}-p_2\right)]$$

$$L_1 = p_1\ [N_T\ \left(\alpha-p_1\right)+\gamma N_B\left(\overline{\alpha}-\alpha\right)]+\prod_2 +$$
$$\delta\,[C-N_T\left(\alpha-p_1\right)-\gamma N_B\left(\overline{\alpha}-\alpha\right)-\theta(1-\gamma)N_B\left(\overline{\alpha}-p_2\right)]\ +\mu\,(\alpha-p_1)$$

To find the subgame perfect equilibrium, we solve the game backwards.

Large Capacity: $C\ >\ C_3\ =\ \alpha N_T/2+N_B[\gamma(\overline{\alpha}-\alpha)+(1-\gamma)\theta\overline{\alpha}]/2$. When capacity is large enough, the constraints in both periods do not bind and thus their coefficients are equal to zero. Optimal prices are given by $p_1^h=\dfrac{\alpha}{2}+\dfrac{\gamma\,N_B(\overline{\alpha}-\alpha)}{2N_T}$ and $p_2^h=\overline{\alpha}/2$.

Intermediate Capacity: $C_2 < C < C_3$, where C_3 is given above and C_2 is computed in the last case. We first derive the first-order condition of the second-period Lagrangian with respect to p_2, which yields $\partial L_2/\partial p_2 = (1-\gamma)N_B\ \theta\left(\overline{\alpha}-2\,p_2\right)+\lambda\ (1-\gamma)\theta N_B=0$.

Since in this intermediate case the constraint is binding, $\lambda>0$ and thus

$$C-N_T\left(\alpha-p_1\right)-\gamma N_B\left(\overline{\alpha}-\alpha\right)-\theta(1-\gamma)N_B\left(\overline{\alpha}-p_2\right)=0.$$

Therefore, the price is represented by

$$p_2=\overline{\alpha}+\frac{N_T\,(\alpha-p_1)+\gamma N_B\left(\overline{\alpha}-\alpha\right)-C}{\theta\ N_B(1-\gamma)}.$$

Substituting this price into the Lagrangian of period 1 yields the following:

$$L_1=p_1\,[N_T\left(\alpha-p_1\right)+\gamma N_B(\overline{\alpha}-\alpha)]-$$
$$\frac{[N_T\,(\alpha-p_1)+\gamma N_B\left(\overline{\alpha}-\alpha\right)-C][N_T\,(\alpha-p_1)+\gamma N_B\left(\overline{\alpha}-\alpha\right)+(1-\gamma)\theta N_B\overline{\alpha}-C]}{\theta\ N_B(1-\gamma)}+\mu\,(\alpha-p_1).$$

Differentiating with respect to p_1 and equating to zero yields the optimal solution for p_1. It is straightforward to check that if $C>N_B\dfrac{\overline{\alpha}}{2}(\theta+\gamma(1-\theta))$, then $\alpha - p_1 > 0$, thus $\mu = 0$, and the optimal price in period 1 is represented by $p_1=\dfrac{\alpha}{2}+\dfrac{\gamma\ N_B(\overline{\alpha}-\alpha)}{2N_T}+\dfrac{\mathrm{S}(C)}{N_T+\theta\,(1-\gamma)\,N_B}$

where $S(C)$ is the capacity shortage; in other words,
$\mathrm{S}(C) = \alpha N_T / 2 + N_B[\gamma(\overline{\alpha} - \alpha) + (1-\gamma)\theta\overline{\alpha}]/2 - C$.

Substituting this expression into the equation for p_2 yields $p_2 = \dfrac{\overline{\alpha}}{2} + \dfrac{\mathrm{S}(C)}{N_T + \theta\,(1-\gamma)\,N_B}$.

<u>Very Low Capacity</u>: $C < C_1 = N_B(\theta + \gamma(1-\theta))\overline{\alpha}/2$, $\mu > 0$, and the constraint is binding; i.e., α - $p_1 = 0$. It follows that $p_1 = \alpha$ and thus the airline does not sell to the tourist segment. Substituting this equation into the expression for p_2 yields the following optimal second-price-period for the low-capacity case: $p_2 = \overline{\alpha} - \dfrac{C}{N_B(\theta + \gamma(1-\theta))}$. Since the airline sells only to the business segment, it follows that $p_1 = p_2$.

<u>Low Capacity</u>: $C_1 < C < C_2$ where C_1 is given above. We next derive the expression for C_2. Note that, when $C = N_B(\theta + \gamma(1-\theta))\overline{\alpha}/2$, the airline sells only to business consumers at price $p_2 = \overline{\alpha}/2$. This generates optimal unconstrained profits from the business segment. However, when the capacity exceeds that threshold, the airline continues to charge the monopoly business price $p_2 = \overline{\alpha}/2$ until capacity is large enough that the benefit revenue from the additional tourist travelers exceeds the losses from deviating from the optimal price for the business traveler segment. The loss in selling to business travelers is given by
$p_1^l N_B[\gamma + \theta(1-\gamma)] - \gamma N_B(\overline{\alpha} - \alpha)p_1^m - (1-\gamma)\theta N_B(\overline{\alpha} - p_2^m)p_2^m$.
The revenue from selling to the tourist segment is given by $p_1^m N_T(\alpha - p_1^m)$. Equating these two equations yields the threshold capacity level C_2:

$$C_2 = \frac{N_T^2\alpha + N_T N_B[\overline{\alpha}(\gamma + \theta - \gamma\theta) - \gamma\alpha] - \sqrt{[N_T(N_T - \gamma N_B)[N_T\alpha^2 - \gamma N_B(\overline{\alpha} - \alpha)^2][N_T + N_B\theta(1-\gamma)]]}}{2N_T}.$$

Thus for any $N_B\dfrac{\overline{\alpha}}{2}[\theta + \gamma(1-\theta)] < C < C_2$, the airline charges $p_1 = p_2 = \dfrac{\overline{\alpha}}{2}$, and for any $C_2 < C$, the airline charges $p_1 = \dfrac{\alpha}{2} + \dfrac{\gamma\,N_B(\overline{\alpha} - \alpha)}{2N_T} + \dfrac{\mathrm{S}(C)}{N_T + \theta\,(1-\gamma)\,N_B}$ and $p_2 = \dfrac{\overline{\alpha}}{2} + \dfrac{\mathrm{S}(C)}{N_T + \theta\,(1-\gamma)\,N_B}$. Note that $\alpha N_T/2 + N_B[\gamma(\overline{\alpha} - \alpha) + (1-\gamma)\theta\overline{\alpha}]/2 > C_2$ for $N_T\alpha^2 \geq \gamma N_B(\overline{\alpha} - \alpha)^2$ and that $N_B\dfrac{\overline{\alpha}}{2}[\theta + \gamma(1-\theta)] < C_2$ when $\dfrac{N_T N_B\gamma\alpha - N_T\alpha^2 + \sqrt{[N_T(N_T - \gamma N_B)[N_T\alpha^2 - \gamma N_B(\overline{\alpha} - \alpha)^2][N_T + N_B\theta(1-\gamma)]]}}{2N_T} > 0.$. If this condition is not satisfied, then the airline moves directly from a policy of low capacity to a policy of intermediary capacity.

Proposition 2:

Large Capacity: C > C₄ = $(\gamma M_B + M_T)\dfrac{4-3\beta}{2(3-2\beta)} + \dfrac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)}$. Let x be the tourist with the highest utility that will purchase the ticket in period 3 at price p_3. What we do next is find the values of x, p_1, p_2, and p_3 that will constitute an equilibrium to the new game. We begin at period 3, where profits are represented by $\pi_3 = M_T\ (x - p_3)\, p_3 / \alpha$. Maximization with respect to the price yields $p_3 = x/2$ and $\pi_3 = M_T\ x^2 / 4\alpha$. In period 2, the expected profits depend on the business segment that did not purchase tickets in period 1, and the expected third-period profits. Thus, second-period profits can be represented as

$$\pi_2 = \frac{p_2\ M_B\ (1-\gamma)\ \theta\ (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} + \pi_3 .$$

First-order conditions imply that the price in period 2 is represented by $p_2 = \bar{\alpha}/2$ and profits are represented by $\pi_2 = \dfrac{M_B(1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \dfrac{M_T\ x^2}{4\alpha}$.

In period 1, profits are represented by

$$\pi_1 = \frac{M_T\ p_1(\alpha - x)}{\alpha} + p_1\gamma M_B + \frac{M_B\ (1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \frac{M_T\ x^2}{4\alpha} .$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This yields the following equation for x: $x - p_1 = \beta\ (x - p_3)$. Substituting $p_3 = x/2$ and solving for x yield: $x = 2p_1 / (2-\beta)$. Substituting this expression for x in the profit function π_1 yields

$$\pi_1 = \frac{M_T\ p_1}{\alpha}(\alpha - \frac{2\,p_1}{2-\beta}) + p_1\gamma M_B + \frac{M_B\ (1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \frac{M_T}{\alpha}\frac{p_1^{\ 2}}{(2-\beta)^2} .$$

The first-order condition for optimality yields the following solution for prices:

$p_1^b = \dfrac{\alpha\,(2-\beta)^2\ (M_T + \gamma M_B)}{2M_T(3-2\beta)}$ and $p_3^b = p_1^b/(2-\beta)$. Note that, since $(2 - \beta) > 1$, it follows that $p_1^b > p_3^b$ and, since $(2-\beta)^2 > (3-2\beta)$, $p_1^b > p_1^h$ where the latter is the unconstrained price of the two-period game. Substituting p_1 into the objective function yields

$\pi_1 = \dfrac{\alpha\,(2-\beta)^2\ (M_T + \gamma M_B)^2}{4M_T(3-2\beta)} + \dfrac{M_B\ (1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)}$. Also, it is easy to see that the derivative of the profits with respect to β yields $\dfrac{\partial \pi_1}{\partial \beta} = -\dfrac{\alpha\,(2-\beta)(1-\beta)(M_T + \gamma M_B)^2}{2M_T(3-2\beta)^2} < 0$.

Demand of the unconstrained case:

$$D_1(p_1) = \frac{M_T}{\alpha}(\alpha - \frac{2p_1}{2-\beta}) + \gamma M_B = (\gamma M_B + M_T)(\frac{1-\beta}{3-2\beta}).$$

$$D_2(p_2) = \frac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)} .$$

$$D_3(p_3) = \frac{M_T x}{2\alpha}\frac{2p_1}{2-\beta} = \frac{M_T}{\alpha(2-\beta)}\frac{\alpha(2-\beta)^2\,(M_T+\gamma M_B)}{2M_T(3-2\beta)} = \frac{(2-\beta)\,(M_T+\gamma M_B)}{2(3-2\beta)}.$$

$$C_4 = \sum_{i=1}^{3} D_i(p_i) = (\gamma M_B + M_T)\frac{4-3\beta}{2(3-2\beta)} + \frac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)}.$$

Thus, for any capacity above this demand ($C_4 < C$) the firm is unconstrained and follows the preceding policy. Define C_3'=Max$\{2\gamma M_B + \frac{(1-\gamma)\,\theta\,\bar{\alpha}M_B}{2(\bar{\alpha}-\alpha)}$, $C_3\}$. When $C_3' < C < C_4$, we have to solve the following problem.

<u>Intermediate Capacity</u>: $C_3' < C < C_4$. Let x be the tourist with the highest utility that will purchase a ticket in period 3 at price p_3. We begin at period 3, where profits are represented by $\pi_3 = M_T\,(x - p_3)\,p_3/\alpha$, s.t. $S_3 - M_T\,(x-p_3)/\alpha \geq 0$, where S_3 is the capacity at the beginning of period 3. When the constraint is binding, the price is given by $p_3 = x - \frac{\alpha S_3}{M_T}$ and the profits by $\pi_3 = (x - \frac{\alpha S_3}{M_T})S_3$. In period 2, the expected profits depend on 1) the business segment that did not purchase tickets in period 1, and 2) the expected third-period profits. Thus, second-period profits can be represented as

$$\pi_2 = \frac{p_2\; M_B\;(1-\gamma)\,\theta\,(\bar{\alpha}-p_2)}{\bar{\alpha}-\alpha} + \pi_3.$$

First-order conditions imply that the price in period 2 is represented by $p_2 = \bar{\alpha}/2$ and profits are represented by $\pi_2 = \frac{M_B(1-\gamma)\,\theta\,\bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + (x - \frac{\alpha S_3}{M_T})S_3$. Note that the remaining capacity at the end of the third period is $S_3 = C - M_T - \frac{xM_T}{\alpha} - \gamma M_B - \frac{(1-\gamma)\,\theta\,\bar{\alpha}}{2(\bar{\alpha}-\alpha)}M_B$.

In period 1, profits are represented by

$$\pi_1 = \frac{M_T\;p_1(\alpha - x)}{\alpha} + p_1\gamma M_B + \frac{M_B(1-\gamma)\,\theta\,\bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + (x - \frac{\alpha S_3}{M_T})S_3.$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This yields that: $x - p_1 = \beta\,(x - p_3)$. Substituting $p_3 = x - \frac{\alpha S_3}{M_T}$ and solving for x yields $x = p_1 + \frac{\beta\alpha S_3}{M_T}$ (note that this analysis is true when $x < \alpha$, which corresponds to $C > 2\gamma M_B + \frac{(1-\gamma)\,\theta\,\bar{\alpha}M_B}{2(\bar{\alpha}-\alpha)}$).

Substituting these expressions for x and S_3 into the profit function π_1 yields

$$\pi_1 = \frac{M_T\ p_1}{\alpha}(\alpha - p_1 - \frac{\beta\alpha S_3}{M_T}) + p_1\gamma M_B + \frac{M_B(1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} +$$
$$\frac{p_1}{1-\beta}[C - M_T + \frac{p_1 M_T}{\alpha} - \gamma M_B - \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)}] -$$
$$\frac{\alpha(1-\beta)}{M_T(1-\beta)^2}[C - M_T + \frac{p_1 M_T}{\alpha} - \gamma M_B - \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)}]^2 \quad .$$

First-order conditions yield the following solution for prices:

$$p_1^c = \frac{\alpha}{2M_T}[(M_T + \gamma M_B)(1-\beta) - (1+\beta)(C - M_T - \gamma M_B - \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)})]$$ and

$$p_3^c = \frac{\alpha}{M_T}[M_T + \gamma M_B + \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)} - C] \ .$$

The profits $\prod^{LMC}$ of the constrained case ($C_3 < C < C_4$) are given by $\prod^{LMC} = \frac{\alpha Z}{4M_T}[4(M_T + \gamma M_B) - Z(3+\beta)] + \frac{(1-\gamma)\ \theta M_B(\bar{\alpha})^2}{4(\bar{\alpha}-\alpha)})]$ where

$$Z = C - \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)} .$$

Next, we compared these profits to the profits from the two-period game where the firm does not offer a last-minute deal and capacity is large ($C_3 < C$):

$$\prod^{2L} = \frac{\alpha}{4M_T}(M_T + \gamma M_B)^2 + \frac{(1-\gamma)\ \theta M_B(\bar{\alpha})^2}{4(\bar{\alpha}-\alpha)})] .$$

$$\prod^{LMC} - \prod^{2L} = \frac{\alpha}{4M_T}[4(M_T + \gamma M_B)Z - Z^2(3+\beta) - (M_T + \gamma M_B)^2] .$$

This term is positive if $Z_1 < Z < Z_2$ where $Z_{1,2} = (M_T + \gamma M_B)\frac{2 \pm \sqrt{1-\beta}}{3+\beta}$. Define

$C_{Zi} = Z_i + \frac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)}$ i={1, 2}. It is clear that, if we show that $C_{Z1} < C_3 < C_4 < C_{Z2}$, then for the relevant range of capacity $C_3 < C < C_4$ the profit for the last-minute-deal is larger than that of the two-period game (without a last-minute deal).

$C_{Z2} - C_4 = (M_T + \gamma\, M_B)\left(\frac{2+\sqrt{1-\beta}}{3+\beta} - \frac{4-3\beta}{2(3-2\beta)}\right)$: It is straightforward to show that this expression is positive since $3\beta^2 - 3\beta + 2(3-2\beta)\sqrt{1-\beta} \geq 0$ is positive for $0 < \beta < 1$.

$C_3 - C_{Z1} = (M_T + \gamma\, M_B)\left(\frac{1}{2} - \frac{2-\sqrt{1-\beta}}{3+\beta}\right)$: It is straightforward to show that this expression is positive since $2\sqrt{1-\beta} + \beta - 1$ is positive for $0 < \beta < 1$.

Also, it is easy to see that the derivative of the profits with respect to β yields

$$\frac{\partial \prod^{LMC}}{\partial \beta} = -\frac{\alpha Z^2}{4M_T} < 0.$$

First period price can be greater or less than the price of the unconstrained two-period game:

Define p_i^c as prices of period *i* in the game we solved in proposition 3 and $p_3^c < p_3^h$ as prices in period i of the two-period unconstrained game (proposition 1, large capacity).
It is straightforward to show that $p_3^c < p_3^h$ if C> C_3.
We still must show that $p_1^c > p_1^h$ if $\beta<\beta^*$. Note that
$p_1^c - p_1^h = \frac{\alpha}{4M_T}[2(M_T + \gamma M_B) - (1+\beta)[2C - \frac{(1-\gamma)\,\theta\,\bar{\alpha} M_B}{\bar{\alpha}-\alpha})]]$, which is positive if
$C < C^* = \frac{M_T + \gamma M_B}{1+\beta} + \frac{(1-\gamma)\,\theta\,\bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)}]$. Note that, when $C_4 < C^*$ then it is always true that $p_1^c > p_1^h$. However, if $C_3 < C^* < C_4$, there then exists a range of capacity $C^* < C < C_4$ such that $p_1^c < p_1^h$. It is easy to show that $C^* - C_3 = \frac{(1-\beta)(M_T + \gamma M_B)}{2(1+\beta)} > 0$.
$C^* - C_4 = \frac{(1-\beta)(2-3\beta)(M_T + \gamma M_B)}{6+2\beta-4\beta^2}$, which is positive for $0 < \beta < \beta^* = 2/3$ and negative for $0 < \beta^* < \beta < 1$.

The solution for the game where the firm chooses the optimal β in the first stage: In this section, we solve the game where the airline decides the frequency of the last-minute deal (i.e., the probability that it introduces the third-period β). We start the analysis with the case of large capacity and continue with the case of smaller capacity.

Large Capacity: C > $C_4 = \frac{2(\gamma M_B + M_T)}{3} + \frac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)}$. Let *x* be the tourist with the highest utility that will purchase the ticket in period 3 at price p_3. What we do next is find the values of *x*, p_1, p_2, p_3, and β that will constitute an equilibrium to the new game. We begin at period 3 where profits are represented by $\pi_3 = M_T\,(x - p_3)\,p_3/\alpha$.
Maximization with respect to the price yields $p_3 = x/2$ and $\pi_3 = M_T\,x^2/4\alpha$.

In period 2, expected profits depend on 1) the business segment that did not purchase tickets in period 1, and 2) expected third-period profits. Thus, second-period profits can be represented as

$$\pi_2 = \frac{p_2\,M_B\,(1-\gamma)\,\theta\,(\bar{\alpha} - p_2)}{\bar{\alpha}-\alpha} + \beta\pi_3.$$

First-order conditions imply that the price in period 2 is represented by $p_2 = \bar{\alpha}/2$ and profits by

$$\pi_2 = \frac{M_B(1-\gamma)\,\theta\,\bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \frac{\beta M_T\,x^2}{4\alpha}.$$

In period 1, profits are represented by

$$\pi_1 = \frac{M_T\ p_1(\alpha - x)}{\alpha} + p_1\gamma M_B + \frac{M_B\ (1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \frac{M_T\ x^2}{4\alpha}.$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x: $x - p_1 = \beta\,(x - p_3)$. Substituting $p_3 = x/2$ and solving for x yields $x = 2p_1 / (2-\beta)$. Substituting this expression for x in the profit function π_1 yields

$$\pi_1 = \frac{M_T}{\alpha(2-\beta)^2}[\alpha(2-\beta)^2 + p_1{}^2(3\beta-4)] + p_1\gamma M_B + \frac{M_B\ (1-\gamma)\ \theta\ \bar{\alpha}^2}{4(\bar{\alpha}-\alpha)}\ .$$

The first-order conditions with respect to p_3 and β yield the following solution for prices:

$p_1^b = \dfrac{\alpha\,(2-\beta)^2\ (M_T + \gamma M_B)}{2M_T(4-3\beta)}$ and $\beta = 2/3$.

Substituting β into the terms of the pricing terms yields $p_3^b = \dfrac{\alpha(M_T + \gamma M_B)}{M_T}$ and

$p_1^b = \dfrac{4\alpha(M_T + \gamma M_B)}{9M_T}$. It is obvious to see that $p_1^b > p_3^b$.

Demand of the unconstrained case:

$$D_1(p_1) = \frac{M_T}{\alpha}(\alpha - x) + \gamma M_B = \frac{(\gamma M_B + M_T)}{3}.$$

$$D_2(p_2) = \frac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)}.$$

$$D_3(p_3) = \frac{M_T(x-p_3)}{2\alpha} = \frac{M_T + \gamma M_B}{3}.$$

$$C_4 = \sum_{i=1}^{3} D_i(p_i) = \frac{2(\gamma M_B + M_T)}{3} + \frac{M_B(1-\gamma)\theta\bar{\alpha}}{2(\bar{\alpha}-\alpha)}.$$

Thus, for any capacity above the demand $C_4 < C$, the firm is unconstrained and follows the preceding policy. Define $C_3' = \text{Max}\{2\gamma M_B + \dfrac{(1-\gamma)\ \theta\ \bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)}, C_3\}$. When $C_3' < C < C_4$, we have to solve the following problem.

Intermediate Capacity: $C_3' < C < C_4$. Let x be the tourist with the highest utility that will purchase a ticket in period 3 at price p_3. What we do next is find the values of x, p_1, p_2, and p_3 that will constitute an equilibrium to the new game. We begin at period 3, where profits are represented by $\pi_3 = M_T\ (x-p_3)\,p_3/\alpha$, s.t. $S_3 - M_T\ (x-p_3)/\alpha \geq 0$, where S_3 is the capacity at the beginning of period 3. When the constraint is binding, price is given by $p_3 = x - \dfrac{\alpha S_3}{M_T}$ and profits by $\pi_3 = (x - \dfrac{\alpha S_3}{M_T})S_3$.

In period 2, expected profits depend on 1) the business segment that did not purchase tickets in period 1, and 2) profits expected in period 3. Thus, second-period profits can be represented as

$$\pi_2 = \frac{p_2\, M_B\, (1-\gamma)\, \theta\, (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} + \beta\pi_3 .$$

First-order conditions imply that the price in period 2 is represented by $p_2 = \bar{\alpha}/2$ and profits are represented by $\pi_2 = \frac{M_B(1-\gamma)\,\theta\,\bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + (x - \frac{\alpha S_3}{M_T})\beta S_3$. Note that the remaining capacity at the end of period 3 is $S_3 = C - M_T + \frac{xM_T}{\alpha} - \gamma M_B - \frac{(1-\gamma)\,\theta\,\bar{\alpha}}{2(\bar{\alpha}-\alpha)} M_B$.

In period 1, profits are represented by

$$\pi_1 = \frac{M_T\, p_1(\alpha - x)}{\alpha} + p_1\gamma M_B + \frac{M_B(1-\gamma)\,\theta\,\bar{\alpha}^2}{4(\bar{\alpha}-\alpha)} + \beta(x - \frac{\alpha S_3}{M_T})S_3 .$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x: $x - p_1 = \beta\,(x - p_3)$. Substituting $p_3 = x - \frac{\alpha S_3}{M_T}$ and solving for x yields

$$x = \frac{\beta\alpha S_3}{2M_T(1-\beta)(\bar{\alpha}-\alpha)}[2M_T p_1(\bar{\alpha}-\alpha) + 2M_T\alpha^2\beta + 2C\alpha\bar{\alpha}\beta - 2M_T\alpha\bar{\alpha}\beta + 2M_B\alpha^2\beta\gamma - M_B\alpha\bar{\alpha}\beta(2\gamma + \theta - \gamma\theta) .$$

Substituting these expressions for x and S_3 into the profit function π_1 and deriving with respect to p_1 yields

$$p_1^c = \frac{1}{2M_T(\bar{\alpha}-\alpha)}\,[\alpha(\bar{\alpha}-\alpha)[M_T - 2C\beta + \beta M_T + \gamma M_B(1+\beta)] + M_B\alpha\bar{\alpha}\beta\theta(1-\gamma)] .$$

Substituting this expression into the profits function and deriving it with respect to β yields

$$\frac{\partial\pi_1(p_1^c)}{\partial\beta} = -\frac{\alpha}{4M_T(\bar{\alpha}-\alpha)^2}\,[(\bar{\alpha}-\alpha)(2c - M_T - \gamma M_B) + M_B\bar{\alpha}\theta(1-\gamma)]^2 < 0 .$$

As the profit decreases with β, the firm chooses β = 0.

<u>Proposition 3</u>:
Define the capacity at the beginning of the i^{th} period as S_i. Recall that, due to uncertainty about business consumers' arrival, the initial capacity at the beginning of the third period can be $S_3=S_2$ if business consumers do not arrive or $S_3=S_2 - \frac{M_B\bar{\alpha}}{2(\bar{\alpha}-\alpha)}$ if business consumers do arrive. We also define x to be the tourist with the highest utility that will purchase a ticket in period 3 at price p_3. Recall that this consumer is indifferent between purchasing a ticket in period 1 at price p_1 and purchasing a ticket during period 3 at price p_3. We next solve the game starting with the highest capacity case.

Large Capacity: C > $C_{34} = M_T \frac{\alpha}{2} + \frac{M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)}$. To find the subgame perfect equilibrium, we solve the game backward starting at period 3, where profits are represented by $\pi_3 = M_T\ (x - p_3)\, p_3 / \alpha$. Maximization with respect to price yields $p_3 = x/2$ and $\pi_3 = M_T\ x^2 / 4\alpha$.
In period 2, the expected profits are given by

$$\pi_2 = \theta[\frac{p_2\ M_B\ \ (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} + \pi_3] + (1-\theta)\pi_3 .$$

Maximization with respect to price yields $p_2 = \bar{\alpha}\ /2$ and $\pi_2 = \frac{1}{4}[\frac{M_T x^2}{\alpha} + \frac{\theta M_B (\bar{\alpha})^2}{\bar{\alpha} - \alpha}]$.
We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x: $x - p_1 = \theta\ (x - p_3) + (1 - \theta)\ (x - p_3)$. Substituting $p_3 = x/2$ and solving for x yields $x = 2\ p_1$. In period 1, the expected profits are given by

$\pi_1 = \frac{p_1\ M_T\ \ (\alpha - x)}{\alpha} + \pi_2$. Maximization with respect to price yields $p_1 = \alpha/2$ and $x = \alpha$. This indicates that tourist consumers do not purchase during the first period and thus the firm sells to business consumers during period 2 at price $p_2 = \bar{\alpha}\ /2$ and to tourist consumers in period 3 at price $p_3 = \alpha\, /2$. This in this case there is no price discrimination within the tourist segment.

High-Intermediate Capacity: $\frac{M_T(\bar{\alpha} - \alpha) + M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)} = C_{23} < C < C_{34}$.

This is the case at which the firm is unconstrained in period 2, and if the business consumers do arrive, the firm will be constrained in period 3. We can consider two cases in period 3: If business consumers do not arrive during the second period, the initial capacity is given by $S_3 = S_2$. In this case, the firm is unconstrained during the third period and profits are represented by $\pi_{31} = M_T\ (x - p_{31})\, p_{31} / \alpha$. Maximization with respect to price yields $p_{31} = x/2$ and $\pi_{31} = M_T\ x^2 / 4\alpha$.

If business consumers arrive during the second period, the initial capacity is given by $S_3 = S_2 - \frac{M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)}$. In this case, the firm is constrained during the third period and has to adjust the third-period price accordingly. Thus, demand at price p_{32} is $\frac{M_T\ \ (x - p_{32})}{\alpha} = S_3$. Solving this equality, we determine that the third-period price is $p_{32} = \frac{xM_T - \alpha[S_2 - \frac{M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)}]}{M_T}$ and

$$\pi_{32} = \frac{[S_2 - \frac{M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)}][\alpha[S_2 - \frac{M_B \bar{\alpha}}{2(\bar{\alpha} - \alpha)}] - M_T\ x]}{M_T} .$$

In period 2, the expected profits are given by $\pi_2 = \theta[\frac{p_2 M_B (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} + \pi_{32}] + (1-\theta)\pi_{31}$.

Maximization with respect to price yields $p_2 = \bar{\alpha}/2$ and

$$\pi_2 = \frac{(1-\theta)M_T x^2}{4\alpha} + \theta[\frac{M_B(\bar{\alpha})^2}{4(\bar{\alpha}-\alpha)} + [S_2 - \frac{M_B\bar{\alpha}}{2(\bar{\alpha}-\alpha)}][x - \frac{[S_2 - \frac{M_B\bar{\alpha}}{2(\bar{\alpha}-\alpha)}]\alpha}{M_T}]].$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x: $x - p_1 = \theta(x - p_{32}) + (1-\theta)(x - p_{31})$. Substituting p_{31} and p_{32} for $S_2 = C - \frac{\alpha - x}{\alpha} M_T$ and solving for x yield: $x = \frac{\theta\alpha[2C(\bar{\alpha}-\alpha) - M_B\bar{\alpha}] + 2M_T(\bar{\alpha}-\alpha)(\theta\alpha - p_1)}{M_T(\bar{\alpha}-\alpha)(1-\theta)}$. In period 1, the expected profits are given by $\pi_1 = \frac{p_1 M_T (\alpha - x)}{\alpha} + \pi_2$. Maximization with respect to price yields $p_1 = \alpha\frac{M_T(\bar{\alpha}-\alpha)(1+\theta) - \theta[2C(\bar{\alpha}-\alpha) - M_B\bar{\alpha}]}{2M_T(\bar{\alpha}-\alpha)}$ and $x = \alpha$. This indicates that tourist consumers do not purchase during the first period and thus the firm sells to business consumers during period 2 at price $p_2 = \bar{\alpha}/2$ and to tourist consumers in period 3 at price $p_3 = \alpha/2$. Just as in the previous case, there is no price discrimination within the tourist segment in this case as well.

<u>Low-Intermediate Capacity</u>: $\frac{\bar{\alpha} M_B}{2(\bar{\alpha}-\alpha)} = C_{12} < C < C_{23}$.

This is the case at which the firm is constrained in period 2, and if the business consumers do arrive, the firm will not have any seats left for the third period. We can consider two cases in period 3: If business consumers do not arrive during the second period, the initial capacity is given by $S_3 = S_2$. In this case, the firm is constrained during the third period and has to adjust the third-period price accordingly. Thus, demand at price p_{31} is $\frac{M_T (x - p_{31})}{\alpha} = S_3$. Solving this equality, we determine that the third-period price is $p_{31} = \frac{xM_T - \alpha S_2}{M_T}$ and $\pi_{31} = \frac{S_2(M_T x - \alpha S_2)}{M_T}$.

If business consumers arrive during the second period, there is no capacity left for the third period; in other words, $\pi_{32} = 0$. In period 2, the firm is also restricted by the initial capacity S_2 and the demand at price p_2, $\frac{M_B (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} = S_2$. Solving this equality, we determine that the second-period price is $p_2 = \frac{\bar{\alpha}(M_B - S_2) + \alpha S_2}{M_B}$. Recall that $S_2 = C - \frac{\alpha - x}{\alpha} M_T$. The second-period expected profits are $\pi_2 = \theta[\frac{p_2 M_B (\bar{\alpha} - p_2)}{\bar{\alpha} - \alpha} + \pi_{32}] + (1-\theta)\pi_{31}$, which yields

$$\pi_2 = \frac{1}{M_T M_B \alpha^2}[\alpha C - M_T(\alpha - x)]\begin{bmatrix} M_T M_B \alpha^2 + M_T[xM_T - \alpha(M_T + M_B)]\theta(\bar{\alpha} - \alpha) - \\ \alpha C[M_B \alpha(1-\theta) + \theta M_T(\bar{\alpha} - \alpha)] \end{bmatrix}.$$

We now use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 1 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x: $x - p_1 = (1-\theta)(x - p_{31})$. Substituting p_{31} and solving for x yields $x = \frac{C\alpha(1-\theta) + M_T[p_1 - \alpha(1-\theta)]}{M_T\theta}$. In period 1, the expected profits are given by

$\pi_1 = \frac{p_1 M_T \ (\alpha - x)}{\alpha} + \pi_2$. Maximization with respect to price yields

$p_1 = \alpha \frac{2C[M_T(\bar{\alpha} - \alpha) + M_B\alpha(1-\theta)] + M_T[2M_T(\bar{\alpha} - \alpha) - M_B(2\alpha - \alpha\theta + \bar{\alpha}\theta)]}{2M_T[M_T(\bar{\alpha} - \alpha) - \alpha M_B]}$. It is easy to verify that $x < \alpha$. Thus, under this capacity range, the firm sells to some tourist consumers during the first period and to other tourist consumers during the third period.

To find out what C_{23} is, note that this case is valid only if S_2 is smaller than the unconstrained demand in period 2. The latter is equal to $\frac{M_B \ \bar{\alpha}}{2(\bar{\alpha} - \alpha)}$. Recall that $S_2 = C - \frac{\alpha - x}{\alpha} M_T$ and substituting x into this equation yields $\frac{M_T(\bar{\alpha} - \alpha) + M_B\bar{\alpha}}{2(\bar{\alpha} - \alpha)} = C_{23}$.

<u>Low Capacity</u>: $C < C_{12}$. If capacity is lower than the unconstrained demand in period 2, ($\frac{\bar{\alpha} M_B}{2(\bar{\alpha} - \alpha)}$), the firm has no incentive to sell tickets in period 1. Since if the business consumers arrive in period 2 it sells in higher price to all of them and if they do not arrive it sells the same seats to tourists in period 3. Thus, in this case, no price discrimination within the tourist segment is possible.

<u>Proposition 4</u>:

We assume that the tourist segment is split into two sub-segments. A fraction $(1-\delta)$ of the M_T consumers arrives at period 0 and has a valuation for the flight that is drawn from a uniform distribution $(0, \underline{\alpha})$, where $\underline{\alpha} < \alpha/2$. The rest of the tourist segment (δM_T) arrives in period one with valuation for the flight that is drawn from a uniform distribution $(\underline{\alpha}, \alpha)$.

We present only the case which is equivalent to the low-intermediate capacity of the previous case (Proposition 5); $C_L \le C \le C_U$, where

$$C_L = \frac{M_B(\alpha - \underline{\alpha})(\bar{\alpha} - \underline{\alpha})(1-\delta) + M_T\delta(\bar{\alpha} - \alpha)[\alpha(1-\delta) - \underline{\alpha}(1-\theta\delta)]}{(\bar{\alpha} - \alpha)[2\alpha(1-\delta) + \underline{\alpha}[2 - \delta(1-\theta)]]} \text{ and}$$

$$C_U = \frac{M_B\bar{\alpha}(\alpha - \underline{\alpha})(1-\delta) + M_T\delta(\bar{\alpha} - \alpha)[\alpha(1-\delta) - \underline{\alpha}[1 + \delta(\delta - 1 - \theta)]]}{(\bar{\alpha} - \alpha)[2\alpha(1-\delta) + \underline{\alpha}[2 - \delta(1-\theta)]]}.$$

Define the capacity at the beginning of the i^{th} period as S_i. Recall that, due to uncertainty about business consumers' arrival, the initial capacity at the beginning of the third period can be $S_3 = S_2$ if business consumers do not arrive or $S_3 = S_2 - \frac{M_B \bar{\alpha}}{2(\bar{\alpha}-\alpha)}$ if business consumers do arrive. We also define x to be the tourist with the highest utility that will purchase a ticket in period 3 at price p_3. Recall that this consumer is indifferent between purchasing a ticket in period 0 at price p_0 and purchasing a ticket during period 3 at price p_3. We only consider the case where $x<\underline{\alpha}$, since if $x>\underline{\alpha}$ the firm does not sell in period 0 and the problem is reduced to the problem we solved in Proposition 5. We consider two cases in period 3: If business consumers do not arrive during the second period, the initial capacity is given by $S_3 = S_2$. In this case, the firm is constrained during the third period and has to adjust the third-period price accordingly. Also, as we are interested in the case were $x<\underline{\alpha}$, then it means that the firm sets the monopoly price in period 1. Thus, demand at price p_{31} is $\frac{(1-\delta)M_T\ (x-p_{31})}{\underline{\alpha}} + \frac{\delta M_T\ (p_1-\alpha)}{\alpha-\underline{\alpha}} = S_3$. Solving this equality, we determine that the third-period price is $p_{31} = \frac{M_T[x(\alpha-\underline{\alpha})(1-\delta)+\delta\underline{\alpha}(p_1-\underline{\alpha})]-S_2\underline{\alpha}(\alpha-\underline{\alpha})}{M_T(1-\delta)(\alpha-\underline{\alpha})}$ and

$\pi_{31} = \frac{S_2 M_T[x(\alpha-\underline{\alpha})(1-\delta)+\delta\underline{\alpha}(p_1-\underline{\alpha})]-(S_2)^2\underline{\alpha}(\alpha-\underline{\alpha})}{M_T(1-\delta)(\alpha-\underline{\alpha})}$. If business consumers arrive during the second period, there is no capacity left for the third period; in other words, $\pi_{32} = 0$.

In period 2, the firm sells only for business consumers and maximizes the following expected profits; $\pi_2 = \theta[\frac{M_B\ p_2(\bar{\alpha}-p_2)}{\bar{\alpha}-\alpha} + \pi_{32}] + (1-\theta)\pi_{31}$ with respect to p_2. If business consumers arrive during the second period, the initial capacity is given by;

$S_2 = C - \frac{(\alpha-p_1)}{(\alpha-\underline{\alpha})}\delta M_T - \frac{(\underline{\alpha}-x)}{\underline{\alpha}}(1-\delta)M_T$. In this case, the firm is constrained during the second period and has to adjust the second-period price accordingly.

Thus $p_2 = (\bar{\alpha}-\alpha)[\frac{\frac{\bar{\alpha}}{\bar{\alpha}-\alpha}M_B - C + \frac{\alpha-p_1}{\alpha-\underline{\alpha}}\delta M_T + \frac{\underline{\alpha}-x}{\underline{\alpha}}(1-\delta)M_T}{M_B}]$.

In period 1 the firm maximizes the following expected profits; $\pi_1 = p_1 \frac{(\alpha-p_1)\ \delta M_T}{\alpha-\underline{\alpha}} + \pi_2$.

This optimization problem yields p_1^*. In period 0, we use the fact that x is the marginal consumer for whom the utility from purchasing the ticket in period 0 is exactly equal to the utility from waiting and buying the ticket in period 3. This fact yields the following equation for x:
$x - p_0 = (1-\theta)(x - p_{31})$. Substituting p_{31} and solving for x yields;
$x = \frac{C\underline{\alpha}(1-\theta)+M_T[p_0(1-\delta)-\underline{\alpha}(1-\theta)]}{M_T\theta(1-\delta)}$. In period 0, the expected profits are given by $\pi_0 = \frac{p_0\ (1-\delta)M_T\ (\underline{\alpha}-x)}{\underline{\alpha}} + \pi_1$. Maximization with respect to price yields p_0^* [1]. It can be shown that $x<\underline{\alpha}$.

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[1] The expressions for p_1^* and p_0^* can be retrieved from the authors by request.