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STABILITY OF AGGREGATION PROCEDURES, ULTRAFILTERS, AND SIMPLE GAMES—A COMMENT

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BATTEAU, BLIN, AND MONJARDET [**1**] define a monotonicity condition which they show to be equivalent to strategy proofness. They also claim that this monotonicity condition is different from the Strong Positive Association Axiom defined by Muller and Satterthwaite [**2**]. Since Muller and Satterthwaite show that Strong Positive Association is equivalent to strategy proofness, monotonicity and Strong Positive Association must be equivalent. The aim of this note is to give a simple and direct proof of the latter equivalence. With this observation, Theorem 1 in Batteau, Blin, and Monjardet follows from the theorem of Muller and Satterthwaite.

I will follow the notation used by Batteau, Blin, and Monjardet. Let d be a voting procedure (or a collective choice procedure). Let π be a preference profile. Let $V_{xy}(\pi)$ denote the set of voters who strictly prefer x to y under profile π.

DEFINITION 1: A voting procedure d is monotonic if

$$\left[x \neq y, d(\pi) = x, V_{xy}(\pi') \supseteq V_{xy}(\pi)\right] \Rightarrow [d(\pi') \neq y].$$

DEFINITION 2: A voting procedure d satisfies Strong Positive Association (SPA) if

$$\left[d(\pi) = x, V_{xy}(\pi') \supseteq V_{xy}(\pi) \text{ for all } y \neq x\right] \Rightarrow [d(\pi') = x].$$

PROPOSITION: *A voting procedure d is monotonic if and only if it satisfies SPA.*

PROOF: Let d be monotonic and let $d(\pi) = x$, $V_{xy}(\pi') \supseteq V_{xy}(\pi)$ for all $y \neq x$. Since d is monotonic, then $d(\pi') \neq y$. Since this is true for all $y \neq x$, then $d(\pi') = x$.

As for the reverse, let d satisfy SPA and $d(\pi) = x$. Let $y \neq x$ and $V_{xy}(\pi') \supseteq V_{xy}(\pi)$. Suppose, a contrario, that $d(\pi') = y$.

Define a new profile π'' by moving x and y to the top of the preferences of π' preserving the internal ranking of $\{x, y\}$ and of the remaining alternatives in each individual's preferences.

Since $d(\pi') = y$ and since d satisfies SPA, it follows that $d(\pi'') = y$. For all z such that $x \neq z \neq y$, $V_{xz}(\pi'') \supseteq V_{xz}(\pi)$ since x is now (at π'') in one of the top two places for all individuals. As for y, $V_{xy}(\pi') \supseteq V_{xy}(\pi)$ (by assumption) and $V_{xy}(\pi'') = V_{xy}(\pi')$ since the position of x relative to y has not changed. Thus we have $d(\pi) = x$, $V_{xw}(\pi'') \supseteq V_{xw}(\pi)$ for all $w \neq x$ but $d(\pi'') \neq x$ which contradicts SPA. *Q.E.D.*

The theorem by Muller and Satterthwaite states that a voting procedure satisfies strategy proofness if and only if it satisfies SPA. Theorem 1 now follows from the above proposition, Muller and Satterthwaite's theorem, and the lemma following Definition 3 in Batteau, Blin, and Monjardet.

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REFERENCES

[1] BATTEAU, P., J. M. BLIN, AND B. MONJARDET: "Stability of Aggregation Procedures, Ultrafilters, and Simple Games," *Econometrica*, 49(1981), 527–34.
[2] MULLER E., AND M. A. SATTERTHWAITE: "The Equivalence of Strong Positive Association and Strategy Proofness," *Journal of Economic Theory*, 14(1977), 412–18.