



An Approach for Determining Optimal Product Sampling for the Diffusion of a New Product

Dipak Jain, Vijay Mahajan, and Eitan Muller

Free samples are an effective means for introducing and promoting a new product. However, product sampling is also expensive. As a result, careful consideration must be given to the question of how many samples should be distributed. To encourage product adoption in any target market, a company needs to determine the "right" amount of sampling. In other words, a firm needs to determine the optimal number of samples that must be available for trial by the innovators, early adopters, and other key consumers who influence the adoption rate of the new product. With too few samples, the product might not reach enough of these key consumers to generate the word-of-mouth recommendations necessary for market success. On the other hand, offering too many free samples is a waste of a company's resources.

Dipak Jain, Vijay Mahajan, and Eitan Muller propose a framework for determining the optimal levels of product sampling. In addition to identifying the upper bounds for the sampling levels of both durable and nondurable products, their model identifies the optimal size of product sampling based on such parameters as the coefficients of innovation and imitation, market potential, discount rate, and gross margin.

Several observations are made regarding the relationships between the optimal sampling level and the various parameters used in the model. For example, a high sampling level is not appropriate for a product with a high coefficient of innovation. On the other hand, if a product has a high coefficient of imitation, the sampling level should be high because a significant number of trials are necessary for word of mouth to be effective. High sampling levels are also indicated by a high discount rate or gross margin.

For durable goods, the optimal level of neutral sampling (i.e., sampling that does not specifically target innovators and early adopters) rarely exceeds 5%, and the maximum level is 7%. The optimal target sampling level is always higher than the corresponding neutral case, but, in most cases, only marginally so. For the parameter ranges chosen in this article, the maximum level for target sampling is approximately 9%. However, it is important to note that the theoretical upper bounds are no more than benchmarks for the maximum possible level of sampling. In practical situations, the optimal level may be considerably lower than these upper bounds. In such cases, the actual values will depend on the values for the various parameters used in the model.

Address correspondence to Dipak Jain, Kellogg Graduate School of Management, Northwestern University, Leverone Hall, 2001 Sheridan Rd., Evanston, IL 60208.

Introduction

Samples are offers of a free amount or a trial of a product for consumers. A sample might be delivered door-to-door, sent in the mail, picked up in a store, found attached to another product, or featured in an advertising offer [12, chap. 23].

Product sampling is one of the most effective ways to introduce a new product [19]. The best way to demonstrate a product's superiority is to get the target customer to try it. Sampling enables a firm to achieve this. Sampling, in fact, offers a firm an effective vehicle to create brand awareness, promote brand identity, improve brand loyalty, and expand product category. By distributing samples to a competitor's customers, sampling also offers a firm an effective offensive mechanism to negate the competitor's promotional programs and to encourage brand switching. Sampling is one of the most widely used consumer-promotional tools. According to a survey conducted in 1990 by Donnelley Marketing Inc., 75% of the major corporations in the survey used sampling as a promotional tool for their new products, and 52% used it for their established products [6]. In fact, after direct consumer coupons and cents-off promotions, new product

sampling was the third most popular consumer-promotional tool used by the survey firms.

Sampling is more effective than other consumer-promotional tools when consumers without direct experience find that verifying the claims of the product is either difficult or risky. Some situations commonly encountered include: (1) a product's features or benefits can not be fully conveyed in advertising (e.g., a unique flavor or aroma in food products and cosmetics) or there are restrictions on how and where a product can be advertised (e.g., ban on broadcast advertising for cigarettes); (2) the product has some new or improved features that can be appreciated to overcome adoption risk only when the product is tested and used by the target customers (e.g., computers, computer softwares, ethical drugs, text books, and cosmetics); and (3) word-of-mouth plays a major role in influencing the product adoption, and hence trial among innovators, early adopters, and other key influencing agents is critical to the success of the product (see [21]).

Despite its advantages, because samples are offered free, sampling is also one of the most expensive ways to introduce a new product. For example, when introducing its new Surf detergent, Lever Brothers distributed free samples to four out of five American households at a cost of \$43 million [12, chap. 23]. One of the earlier most expensive sampling programs for a new product in the U.S. was undertaken by S.C. Johnson & Sons in 1977. To introduce Agree cream rinse to prospective customers, the company distributed more than thirty-one million free samples of the product [25]. It has been estimated that of the \$2.5 billion spent annually by the cigarette industry on promotion, including media advertising, 7%–8% of its is invested in sampling [13]. A recent article published in *Business Week* has discussed the importance of sampling in the software industry. A personal-finance software firm promised to give free samples of the programs to the first one million customers who agreed to pay the shipping charges as part of their new product introduction strategy [7].

Given the high costs associated with sampling, the following two questions need to be considered in the development of cost-effective product sampling plans: who should be offered a free sample of the product? How many samples should be distributed?

Companies have attempted to deal with the first question by avoiding the shotgun approach and identifying the target market by having the interested customers write in for samples or pick them up in a store.

BIOGRAPHICAL SKETCHES

Dipak Jain is the Sandy and Morton Goldman Professor in Entrepreneurial Studies, and Professor of Marketing at the Kellogg Graduate School of Management, Northwestern University. His research interests focus on the areas of market segmentation and competitive market structure analysis, new product diffusion and forecasting models, and choice models for analyzing the impact of marketing mix variables on consumer purchase behavior using scanner panel data. He is an associate editor of the *Journal of Business and Economic Statistics*, the *Journal of Management Science*, and area editor of *Marketing Science*. He is also a member of the editorial board of the *Journal of Marketing* and the *Journal of Marketing Research*.

Vijay Mahajan is the John P. Harbin Centennial Chair in Business, Graduate School of Business, The University of Texas at Austin. He has written extensively on new product diffusion, marketing strategy, and marketing research. He serves on the editorial review boards of several journals including *Journal of Marketing*, *Journal of Marketing Research*, *Marketing Science*, and *Journal of Product Innovation Management*. He currently serves as the editor of the *Journal of Marketing Research*.

Eitan Muller is a Professor of Marketing and Associate Dean at the Recanat Graduate School of Business Administration, Tel Aviv University, Tel Aviv, Israel. He has published extensively on new product diffusion and currently serves on the editorial boards of *Marketing Science* and the *Journal of Marketing Research*.

In addition, some commercial vendors have also developed "selective targeting" approaches to identify the likely customers of a product for promotional purposes (e.g., Carol Wright Share Force Program offered by Donnelley Marketing, see [23]). Although pretest new product forecasting models such as TRACKER [3] and ASSESSOR [26] study the sensitivity of product sampling on the early sales of new consumer-packaged goods, to the best of our knowledge, no guidelines or approaches have been suggested to deal with the second question for products (both durables and nondurables) for which word-of-mouth plays a major role in influencing the product adoption. Examples of such products include new products that try to establish their own market niches (e.g., products that are designed for certain target markets, such as computer software), are new to the market (e.g., picture telephones), possess patent protection (e.g., ethical drugs), or have enough lead time over their competitors due to product technology or unique features to establish themselves in the marketplace.¹ In all these situations a firm enjoys a monopolistic situation for a while before facing competition. For these types of products it is important to determine the optimal number of product samples to generate trial among innovators, early adopters, and other key influencing agents who are critical to the success of the product.²

The objective of this article is to investigate this question. Our underlying thesis is that in any target market the "right" amount of sampling is required to initiate the adoption of a product. Too little sampling may not generate enough trials to initiate the diffusion process for the product, and too much sampling may be a waste of the firm's resources. We use a diffusion modeling approach to investigate this problem [24].

The organization of this article is as follows. The following section delineates the necessary analytical formulations. Based on these analytical formulations, simulation results are presented next. The article con-

cludes with limitations and further extensions of the results and the underlying approach.

Analytical Formulations

The modeling framework determining the optimum levels of product sampling builds on the dynamics of the innovation diffusion process, the process by which an innovation is communicated through certain channels among the members of a social system [24]. The underlying behavioral theory in the development of the analytical model is that the innovation is first adopted by innovators, who, in turn, influence other members of the system (through word of mouth) to adopt it. In other words, the activation of interpersonal communication in a social system is initiated by the innovators and they play a major role in influencing the rate of new product acceptance in the marketplace.

In recent years, a number of mathematical models have been proposed to capture the communication dynamics of the diffusion process and unfold the S-shaped nature of its adoption curve. The Bass model [1], the most popular model in marketing, for example, describes the diffusion process by the following differential equation:

$$\begin{aligned} n(t) &= \frac{dN(t)}{dt} = \left(a + \frac{b}{m} N(t) \right) (m - N(t)) \\ &= a(m - N(t)) + \frac{bN(t)}{m} (m - N(t)) \end{aligned} \quad (1)$$

and

$$N(t = 0) = N_0 \quad (2)$$

where $N(t)$ denotes the cumulative number of adopters by time t , $dN(t)/dt$ or $n(t)$ gives the rate of change in the cumulative number of adopters at time t or the noncumulative number of adopters at time t , m represents the market potential or the number of potential adopters, and a and b denote the coefficients of innovation (external influence) and imitation (internal influence), respectively.³ The term $a(m - N(t))$ defines adoptions due to external influence (e.g., advertising, promotion, etc.) and $bN(t)(m - N(t))/m$ defines adoptions due to the word-of-mouth influence.

¹ In the November 22, 1993 issue of Business Week [4], there is an advertisement from Wordperfect emphasizing the importance of word-of-mouth.

² Our observation is also based on a small survey of ten firms that extensively use product sampling. The survey included four consumer-packaged firms, one computer company, one publishing firm, two advertising agencies, one consulting firm active in the pharmaceutical industry, and one consulting firm active in the consumer-packaged industry. Although respondents from these firms confirmed the importance of the problem, they indicated that they were not aware of any empirical or analytical approaches that deal with this issue. No discussion could be found in recent textbooks on sales promotion such as [2].

³ In fact, as shown by Mahajan et al. [17], the coefficient a also measures the proportion of innovators (as compared to other four adoption categories: early adopters, early majority, late majority, and laggards defined by Rogers [24]).

Equation (2) represents the initial value condition indicating the number of adopters N_0 at the beginning of the diffusion process, i.e., at $t = 0$. Traditionally, the value for N_0 has been assumed to be zero. We postulate that product sampling can be an effective way to create an initial pool of "adopters," and this pool along with the regular group of innovators will influence other potential adopters via word-of-mouth. In other words, the rationale behind creating the initial pool is to enhance the rate of product adoption.

It should be noted that in recent years several attempts have been made to study the impact of other marketing mix variables on the diffusion of a new product. (For a comprehensive review see [15].) Examples included pricing [10], advertising [9], distribution [11], and detailing [14]. Despite such efforts, to the best of our knowledge no efforts have been made to study the impact of sampling on the diffusion of a new product. This is also confirmed by the comprehensive sales promotion literature provided by Blatberg and Neslin [2].

Defining $F(t) = N(t)/m$, $f(t) = n(t)/m$ and $F_0 = N_0/m$, equations (1) and (2) can be restated in terms of the proportion of potential adopters. That is,

$$f(t) = \frac{dF(t)}{dt} = (a + bF(t))(1 - F(t)) \quad (3)$$

and

$$F(t = 0) = F_0 \quad (4)$$

Using the initial condition (4), solution of the differential equation (3) can be derived to show the relationship between sampling, F_0 , adoption curves, $F(t)$ and $f(t)$, and their values, $F(t^*)$ and $f(t^*)$, at the peak time t^* (i.e., point of inflection of S-shaped curve $F(t)$ or peak of $f(t)$). The expressions for $F(t)$, $f(t)$, t^* , $F(t^*)$, $f(t^*)$ are presented in Exhibit 1.

The quantity

$$\frac{1}{a + b} \ln \frac{1 + \left[\frac{b}{a} \right] F_0}{1 - F_0}$$

in the second term on the right-hand side of expression (C) for t^* in Exhibit 1 represents the change that would occur in the time to peak because of the number of adopters at the beginning of the diffusion process generated through product sampling. Note that since $(1 + (b/a)F_0)/(1 - F_0)$ is greater than 1, t^* is reduced by this quantity. It is clear that the larger F_0 is, the larger will be the reduction in t^* .

Exhibit 1. Relationships Between Sampling Level F_0 and Adoption Patterns for the Bass Model

Cumulative proportion of adopters at time t :

$$F(t) = \frac{1 - \frac{a(1 - F_0)}{(a + bF_0)} e^{-(a + b)t}}{1 + \frac{b(1 - F_0)}{(a + bF_0)} e^{-(a + b)t}} \quad (A)$$

Proportion of adopters at time t :

$$f(t) = \frac{\frac{(1 - F_0)}{(a + bF_0)} (a + b)^2 e^{-(a + b)t}}{\left(1 + \frac{b(1 - F_0)}{(a + bF_0)} e^{-(a + b)t} \right)^2} \quad (B)$$

Peak time:

$$t^* = -\frac{1}{(a + b)} \ln \frac{a + bF_0}{b(1 - F_0)} = \frac{1}{(a + b)} \ln \frac{b}{a} - \frac{1}{(a + b)} \ln \frac{1 + \left(\frac{b}{a} \right) F_0}{1 - F_0} \quad (C)$$

Cumulative proportion of adopters at t^* :

$$F(t^*) = \frac{1}{2} - \frac{a}{2b} \quad (D)$$

Proportion of adopters at t^* :

$$f(t^*) = \frac{(a + b)^2}{4b} \quad (E)$$

Expressions (A)–(E) in Exhibit 1 show that F_0 effects $F(t)$, the cumulative proportion of adopters; $f(t)$, the noncumulative proportion of adopters; and t^* , the peak time of the adoption curve. It does not effect $F(t^*)$ and $f(t^*)$, the cumulative and noncumulative proportions of adopters at the peak time, respectively. In other words, it shifts the adoption curve to achieve the same level of adoption earlier in time thereby accelerating product diffusion.

Relationship Between Diffusion and Sampling

As discussed earlier, product sampling is used to demonstrate a product's superiority and to get a potential customer to try the new product. In the innovation diffusion context, the objective of the product sampling is to initiate the diffusion process and to influence the adoption curve. In the Bass diffusion model, this characteristic of the diffusion process is captured by equation (4) that specifies the fraction of adopters at the beginning of the diffusion process and hence

defines the fraction of individuals who adopt the product before the initiation of the diffusion process. The question now is how does product sampling, i.e., F_0 in equation (4), influence the diffusion process.

The dynamics of the product sampling are depicted in Figure 1. As Figure 1(A) shows, as the size of the product sampling is increased (i.e., F_0 is increased from zero to 12%), it helps to achieve the peak earlier.

Note that in the model formulation summarized in Exhibit 1, product sampling F_0 has no impact on the coefficients (i.e., a and b) of the diffusion model. We define such sampling as *neutral sampling*. The main objective in neutral sampling, therefore, is to shift the adoption curve to achieve the same level of penetration or the adoption level earlier in time. Neutral product sampling generally will be observed in those product sampling cases where samples are not targeted to opinion leaders, innovators, or early adopters. This strategy may be used by firms (e.g., text book publishers, pharmaceutical, or cosmetic firms) that are interested in accelerating the initial adoption and can not wait to fully exploit the word-of-mouth dynamics.

One may also argue that because sampling is expensive for firms, it may not be economical to give free samples to every potential adopter. This raises

another question—who should be given the free samples? One possibility may be to target the samples to a specific group—e.g., opinion leaders, innovators, or early adopters. The rationale behind this type of *target sampling* is to create a seed of innovators/change agents who, through word-of-mouth communication, will create additional adopters of the product. Hence, *target sampling influences the adoption curve by generating additional innovators/opinion leaders and compliments the coefficient of innovation, a* . Therefore, in order to capture sampling effect on the coefficient of innovation, we represent this coefficient as a function of the sampling level F_0 , i.e.,

$$a = \phi(F_0) \tag{5}$$

The functional relationship ϕ between a and the product sampling level F_0 may be linear or may exhibit diminishing marginal returns with increasing F_0 . One specification for $\phi(F_0)$ may be the formulation used in the diffusion literature to capture the effects of marketing mix variables such as advertising [9]:

$$\phi(F_0) = \alpha + \beta \log(1 + F_0) \tag{6}$$

where β measures the impact of sampling on the coefficient of innovation and α is another parameter that equals the coefficient of innovation a when there is no sampling. The logarithmic specification ensures diminishing marginal returns of product sampling. It is conceptually appealing and its empirical performance has been examined by [9].

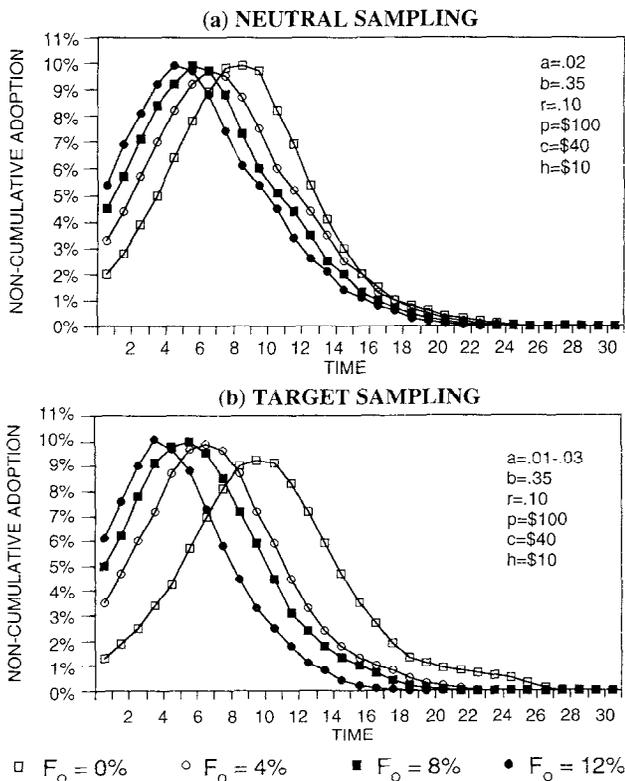
Substitution of equation (6) in equation (3) and further expansion of terms yields:

$$\begin{aligned} f(t) &= \frac{dF(t)}{dt} \\ &= \alpha(1 - F(t)) + \beta \log(1 + F_0)(1 - F(t)) \\ &\quad + bF(t)(1 - F(t)) \end{aligned} \tag{7}$$

If $\beta = 0$, $a = \alpha$, equation (7) reduces to equation (3) and hence, as discussed earlier, it captures neutral sampling (with the initial condition given by equation (4)). The term $\beta \log(1 + F_0)(1 - F(t))$ in equation (7), therefore, captures the impact of sampling on the growth of the product. This impact on the potential adopters is captured by $\beta \log(1 + F_0)$. In this respect, target sampling complements adoptions created by other external sources of influence (i.e., $\alpha(1 - F(t))$).

Note from expressions (A) through (E) in Exhibit 1 that as the targeted product sampling F_0 influences the coefficient of innovation in the Bass model, in addition to the cumulative and noncumulative fraction of

Figure 1. Impact of product sampling on innovation diffusion patterns.



adopters, it also impacts the peak of the adoption curve (i.e., t^* , $f(t^*)$ and $F(t^*)$) given respectively by expressions (C) through (E). Figure 1(B) graphically shows such influences. As depicted in Figure 1(B), an increase in the size of the product sampling influences the rate of adoption as well as the peak of the adoption curve. Target sampling accelerates the diffusion process resulting in a higher peak earlier in time than when there is no sampling. That is, by targeting the sampling to a group of innovators and early adopters, both the rate of adoption and the timing and the peak of the adoption curve can be influenced.

Determination of Optimal Product Sampling Size

The objective of sampling is to initiate the diffusion process. As discussed earlier, too little sampling may not generate enough adoptions to initiate the diffusion process for the product, and too much sampling may be a waste of the firm's resources. The question, therefore, is what should be the optimal size of the product sampling? We suggest here a discounted cash flow framework to answer this question.

To appreciate the formulation, note that if the firm is not engaged in any sampling, its discounted cash flow (net present value) is given by:

$$\pi = \int_0^\infty e^{-rt}(p - c)(dN(t)/dt)dt \quad (8)$$

where p and c are the price and cost of the product, r is the discount rate, $N(t)$ follows equation (1) and at time zero, $N_0 = 0$.

Consider first the case of neutral sampling where the firm offers product samples and decides about an optimal level of N_0 . The cost of handling the sample is h per unit. This includes labor and material cost of wrapping, shipping and handling the sample. It does not include the cost of production (c). We also assume that the firm does not charge any price for the sample—i.e., the product is a giveaway.

The problem, therefore, is to find an optimal N_0 so as to maximize:

$$\pi = \int_0^\infty e^{-rt}(p - c)(dN(t)/dt)dt - (h + c)N_0 \quad (9)$$

where $dN(t)/dt$ is given by equation (1) for neutral sampling.⁴ Substitution of the value of the coefficient

of innovation from equation (6) in equation (1) yields $dN(t)/dt$ for target sampling. It should be noted that the modeling framework assumes $N_0 \leq m$. In other words, we assume that m includes everybody in the market who is a potential customer; if this is not true, then our model needs to be extended to incorporate the cost of sending a free sample to a person who is not a potential customer of the product. Furthermore, we assume that each potential customer receives only one free sample of the product.

The Bass model, equation (1), has been developed to capture the adoption dynamics of a durable-type innovation. There are nondurable products (such as ethical drugs) in which although word-of-mouth plays a critical role in generating the first-purchase adoption curve, the overall success of the product depends upon repeat adoptions. For such products, it is, therefore, important to determine the optimal size of product sampling by simultaneously considering its influence in generating the first-purchase adoptions as well as subsequent repeat adoptions. Following [14] and [18], if we assume that for a nondurable product the total adoptions at any time t are composed of first-time adopters, given by equation (1), and a certain percentage w of the total number of current adopters, equation (9) can be extended to determine optimal product sampling for nondurable products. In this case, the firm maximizes the net present value given by

$$\pi = \int_0^\infty e^{-rt}(p - c)\left(\frac{dN(t)}{dt} + wN(t)\right)dt - (h + c)N_0 \quad (10)$$

When $w = 0$, equation (10) reduces to equation (9). Again, in equation (10), $dN(t)/dt$ is given by equation (1) for neutral sampling and substitution of equation (6) in equation (1) gives $dN(t)/dt$ for the target sampling. For clarity in presenting the results, hereafter, we consider determining an optimal value of $F_0 (= N_0/m)$.

Although equations (9) and (10) specify the optimization formulation to determine the optimal size of product sampling for both durable and nondurable products, respectively, they cannot be integrated to generate explicit closed-form formulae to determine F_0 . They, however, can be solved numerically to find the optimal values of F_0 for given values of p , c , w , h , r , a , (or α and β in equation (6)), b and m in equations (9) and (10). Assuming various values of these parameters, we solve the equations numerically in the next section to obtain some flavor about the nature and role of product sampling on the diffusion process. As

⁴ It should be noted that because equation (8) does not involve N_0 , maximizing equation (9) with respect to N_0 is equivalent to maximizing equation (9) less equation (8).

shown in the Appendix, however, we can derive upper bounds for the size of F_0 . These upper bounds are summarized in Table 1 and specify the maximum sampling a firm should do for a given product. For example, for a durable-type innovation, $F_0 < F(t^*)$ in neutral sampling and $F_0 < F(t^*) + \beta/2b$ for target sampling, where $F(t^*)$ is given in Exhibit 1. Similarly, for nondurable product $F_0 < F(t^*) + w/2b$ for neutral sampling and $F_0 < F(t^*) + (w + \beta)/2b$ for target sampling.

To sum, for a given innovation, Table 1 can be used to ascertain the maximum product sampling that a firm should do. In addition, equation (9) or equation (10) can be solved numerically to obtain the optimal size of product sampling.

Numerical Analysis and Results

In order to examine the effect of product sampling on the diffusion process, we numerically solve equations (1), (2), (9) and (10) to determine the optimal levels of F_0 under a set of values for the parameters a , b , m , r , p , c , h , and w . The numerical procedure involves solving the system dynamics equations, i.e., equations (1) and (2), and the equations representing the NPV value for the firm, i.e., equations (9) and (10). For each given set of parameter values we first solve equation (1) using the Euler–Cauchy numerical method [5] on Lotus 1-2-3. We select a finite time horizon of 30 periods. This time horizon is long enough for the cumulative sales to achieve its maximum and for the noncumulative sales to reach a zero level. We then compute the net present values using the discrete form of equation (9):

$$\sum_{t=0}^{30} \frac{1}{(1+r)^t} \left\{ (p-c) \frac{dN(t)}{dt} \right\} - (h+c) N_0 \quad (11)$$

for each given set of the parameter values and a chosen value for N_0 .

For numerical analysis, the base set of parameter

Table 1. Upper Bounds for Sampling Level F_0 ^a

Sampling Type	Durable Product	Non-Durable Product
Neutral	$F_0 < F(t^*)$	$F_0 < F(t^*) + w/2b$
Target	$F_0 < F(t^*) + \beta/2b$	$F_0 < F(t^*) + (w + \beta)/2b$

^a $F(t^*)$ is the level of penetration when adoptions reach a maximum at time t^* ; w is the repeat rate; β is the responsiveness of the coefficient of innovation to target sampling; b is the coefficient of imitation; m is the market potential.

values chosen are: $a = 0.02$, $b = 0.35$, $m = 54$ million, $r = 0.10$, $p = \$100$, $c = \$40$, and $h = \$10$. The values for the diffusion parameters a , b , and m are the averages of the values of these parameters for the eleven consumer durables analyzed by Bass [1].⁵ The other parameter values were conveniently selected for illustration purposes; however, we vary them over a reasonable range in our analysis.

The optimal level of product sampling is the value of $F_0 (= N_0/m)$ at which the NPV achieves the maximum value. Using the previously stated set of parameter values, we obtained the results shown in Table 2. Based on Table 2, the optimal level of sampling in the neutral case is 3%. Performing a similar analysis for target sampling, we find the optimal level to be 4%.

We also obtain the optimal levels of neutral and target sampling by varying each parameter one at a time over a suitable range. A summary of these results is provided in Table 3.

Figure 2 provides a feel for how sensitive the optimal levels of product sampling are to the changes in the values of the diffusion parameters (coefficients of innovation and imitation), discount rate, and the gross margin. Figure 2 suggests the following:

1. If the coefficient of innovation is high, it is not optimal to have a high sampling level. Given the high value for the coefficient of innovation, there would be enough innovators for the product to take-off, and any attempt to generate additional innovators through sampling may be a waste of resources.
2. If the value of coefficient of imitation is high, the sampling level should be high because high sampling levels would induce significant trial for the product that is essential for the word-of-mouth to have its effect. However, beyond a certain value of the imitation coefficient, the sampling level stabilizes and does not increase any more.
3. If the discount rate is high, then the sampling level should also be high. However, for neutral sampling, the optimal level of sampling becomes constant after certain value of the discount rate.
4. If the gross margin (i.e., (price–cost of production)/price) is high, then the sampling level should also be high in order to induce trial and subsequently generate word-of-mouth effect.

⁵ In the case of target sampling, the value for the coefficient of innovation, a , is obtained from equation (6) by using the average values for the parameter α and β reported in [9]. The chosen values of α and β are $\alpha = 0.013$ and $\beta = 0.0134$.

Table 2. Optimal Sampling Level

F_0 (in %)	0	1	2	3 ^a	4	5	6	7
NPV (in mil.):	142.9	144.5	145.3	145.5	145.1	144.3	143.2	141.7

^a Optimal NPV.

Table 3 reveals the following results:

1. In the case of durables, the optimal level of neutral sampling rarely exceeds 5%, the maximum level being 7%.
2. The optimal target sampling level is always higher than the corresponding neutral case, but in most cases, it is only marginally higher. For the chosen ranges of the parameter values, the maximum level seems to be around 9%.

In Table 4, we report the optimal levels of sampling for nondurable products. These levels have been obtained for a fixed set of values for $a, b, m, r, p, c,$ and h . Our objective here is to examine the relationship between the repeat purchase rate and the optimal level of product sampling. We find from Table 4 that for both neutral and target samplings the optimal level increases with an increase in the value for the repeat purchase rate. This was also the case when we varied the parameters over a suitable range as in the case of durables. For the values reported in Table 4, we find that the correlation between the repeat purchase rate and the product sampling level is 0.997 for both neutral and target sampling. This relationship is further depicted in Figure 3 where we find an almost perfect linear relationship between repeat purchase rate and product sampling.

Furthermore, we see from Table 4 that as the value for the repeat purchase rate increases the optimal level of sampling approaches the repeat purchase rate value. This implies that for a nondurable product, the optimal

sampling level seems to be at least equal to the repeat purchase rate for that product.

It would be interesting to compare the optimal levels of sampling for durables reported in Table 3 with the upper bound values presented in Table 2. For the chosen values of a and b we find by using the expression for $F(t^*)$ in Table 1 that the upper bound for sampling level F_0 is 0.215 for neutral sampling and 0.235 for target sampling. The maximum value of F_0 obtained for the set of values considered in the numerical analysis for durables is 9% implying that the value of F_0 is bounded by the theoretical values provided in Table 1. We find that the same result also holds for nondurable products.⁶ Hence, though the theoretical upper bounds provide a benchmark for the maximum possible level of sampling, in practical situations, the optimal level may be considerably lower than these upper bounds depending upon the values for the various model parameters.

Illustration

Having presented the analytical and simulation results, a question now is how can one use these in a practical setting. Consider, for example, a product manager for a software firm who is interested in knowing the optimal level of sampling for a new software in the United Kingdom.⁷ The implementation of the model will involve the following steps:

Step 1: *Identification of the diffusion parameters—coefficient of innovation, coefficient of imitation, and market potential.* This may be obtained by analyzing the historical data on the diffusion of PCs in the U.K. In fact, studying the diffusion of DOS-based PCs in the market, Givon et al. [8] report that there were 5.12 million users of PCs and the coefficients of innovation (a) and imitation (b) were 0.004 and 0.379 respectively.

Table 3. Ranges of Product Sampling for Durables

Parameters	Range	Neutral Sampling	Target Sampling
Coefficient of imitation	.1-.5	0%-3%	0%-5%
Coefficient of innovation	.005-.03	0%-7%	0%-7%
Discount rate	.08-.18	2%-4%	4%-7%
Price	\$50-\$200	0%-7%	0%-9%
Production costs	\$20-\$70	0%-7%	0%-9%
Handling costs	\$0-\$30	0%-4%	3%-6%

⁶ The estimated values for the other upper bounds for nondurable products are 0.715 and 0.734 for neutral and target sampling respectively, assuming the value of w as 0.35.

⁷ McKenna [20] provides an example about Apple giving samples of *Mac* to influential Americans' months before launching it.

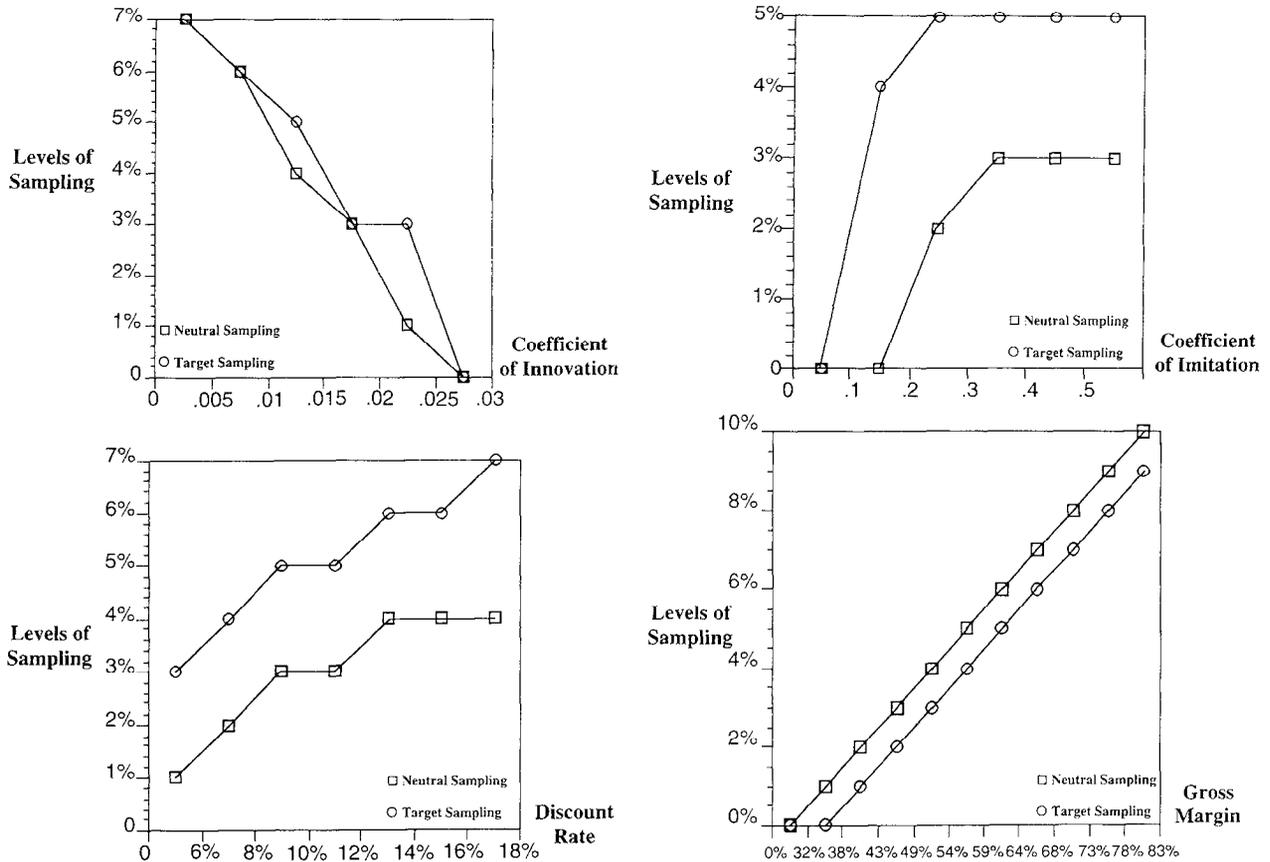


Figure 2. Parameter sensitivity of optimal levels of sampling for durables.

Step 2: *Specification of the discount rate.* A commonly used value for the discount rate is 10%, although one may assume a higher or lower value depending on the economic condition.

Step 3: *Determination of the price of the product.* It is a part of the firm's marketing plan as price is one of the key marketing-mix variables. For example, for the illustration we assume the price of the new software to be about \$250.

Step 4: *Estimation of the product's unit cost and handling cost.* The unit cost of producing the product is available to the firm from its internal records. For the particular software discussed here, the unit cost is assumed as \$83. The handling cost can be easily estimated based on the cost of wrapping and shipping the sample. Firms over time develop a good estimate of this cost, and in the present context it was assumed to be about \$25.

Step 5: *Substitution of the various parameter values in the analytical formulation to obtain the optimal sampling levels.* Using the set of parameter values stated in the previous steps and holding the

effects of other marketing-mix variables (e.g., price, advertising) constant, the results obtained for the optimal sampling level for the U.K. market are presented in Table 5. Based on net present value figures (equation (9)), Table 5 indicates that the optimal level of sampling is 8% for neutral sampling and 7% for target sampling. The proposed analytical framework, therefore, provides a systematic way for obtaining the product sampling levels rather than choosing an arbitrary value based on a certain ad-hoc procedure.⁸

Summary

In this article we have provided an analytical framework that enables one to assess the impact of product sampling on the diffusion of new products, both durables and nondurables where word-of-mouth plays a major role in influencing the product adoption and

⁸ For target sampling, the chosen values of α and β in equation (6) were chosen based on the average values reported in [9].

Table 4. Optimal Levels of Sampling for Nondurables^a

Repeat Rate (w)	Neutral Sampling	Target Sampling
0%	3%	5%
5%	9%	11%
10%	14%	17%
15%	21%	22%
20%	26%	28%
25%	32%	33%
30%	37%	38%
35%	42%	43%
40%	46%	48%
45%	52%	52%
50%	56%	57%
55%	60%	61%
60%	64%	64%
65%	68%	68%
70%	72%	72%

^a The values for the other parameters are: $a = .02$, $b = .35$, $m = 54$ Mil, $r = 0.10$, $p = \$1.00$, $c = \$0.40$, $h = \$0.10$.

hence trial among innovators, early adopters, and other key influencing agents is critical to the success of the product. Our analysis has focused on determining the optimal size of product sampling before a new product is launched in the marketplace and where a new product enjoys a monopolistic situation for a while before facing competition. Given that product sampling is an expensive way to introduce a new product, we use a discounted cash flow framework that incorporates various costs associated with the distribution of free samples.

We derive the optimal levels of product sampling under two cases: (1) neutral sampling—i.e., when the distribution of free product samples only shifts the adoption curve such that a certain level of penetration is attained earlier than the time it would take in the absence of any product sampling; (2) target sampling—i.e., when the free samples are distributed directly to influence the diffusion parameters, especially the coefficient of innovation. In this case, product sampling not only accelerates the rate of adoption but also leads to higher peak sales. These two types of sampling cover a wide variety of situations a product manager may face while introducing new products.

Our results clearly indicate the relationship between levels of sampling and the values of the coefficients of innovation and imitation, the discount rate, and the gross margin. Based on a given set of parameter values, and holding the effects of marketing-mix variables constant, the numerical results suggest that for

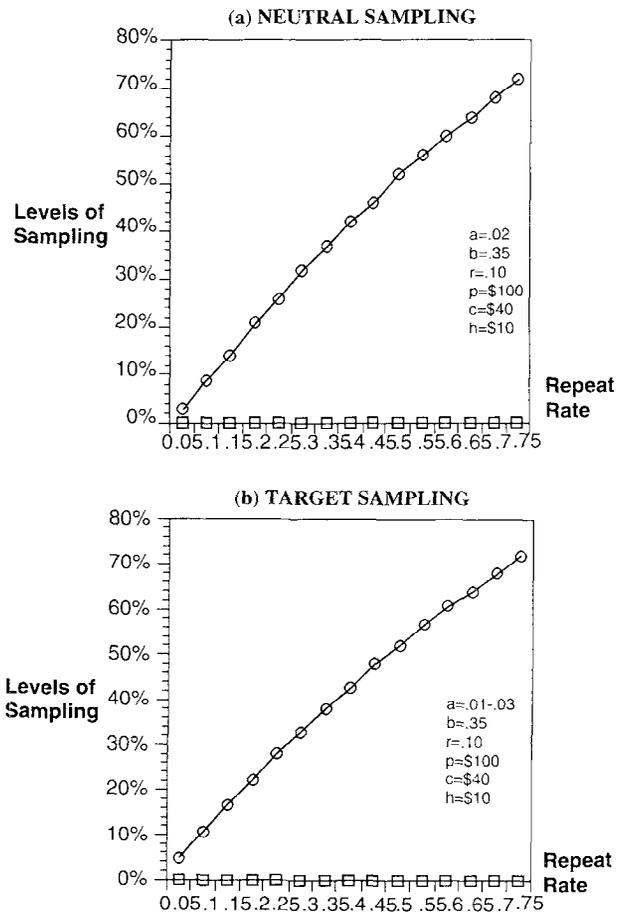


Figure 3. Relationship between repeat rate and product sampling for nondurables.

durables the optimal sampling levels are not more than 9% of the total market potential. As a “benchmark” level, one may choose a value in the 5%–7% range. For nondurables, we find that for “small” values of repeat purchase rate, the optimal level of sampling is greater than the repeat rate. However, as the repeat rate increases, the optimal sampling level becomes almost equal to the repeat purchase rate for the product.

This study is not without limitations. For example, we have not incorporated the effect of competition. This can be studied using a game-theoretic framework. Future research may be directed toward addressing the issue of competition in analyzing the impact of product sampling. In our analysis we have assumed that the samples are distributed free. If the firm charges a certain price for the sample, it is possible to incorporate it in our framework and derive the optimal levels of sampling.

In order to obtain the optimal levels we have as-

Table 5. Optimal Sampling Levels for the U.K. Market

F_0 (in %)	0	1	2	3	4	5	6	7	8	9	10
NPV (Neutral)	251.2	291.8	315.1	330.1	340.2	346.9	351.8	353.4	354.2 ^a	353.8	352.3
NPV (Target)	323.1	342.7	355.6	364.3	370.0	373.5	375.3	375.6 ^b	374.8	373.1	370.5

^a Optimal sampling level under neutral sampling.

^b Optimal sampling level under target sampling.

sumed that sales of the product evolve according to a Bass model. Although the diffusion model [1] has been found appropriate for many product categories, recently several extensions of this model have been suggested in the literature that incorporate marketing-mix variables [10]. It may be of interest to derive the optimal sampling levels while incorporating the effects of marketing-mix variables, as it would provide additional insights into the interactive effects of product sampling and other marketing-mix variables on the diffusion of new products. Such an analysis can be performed within the proposed framework by replacing the system dynamics equation (1) by the alternative model formulation or incorporating effects of these variables in equation (7). Further, other growth models, e.g., Gompertz [16], can also be easily analyzed using our framework.

For certain products a firm is not able to use some effective advertising vehicles such as television (e.g., cigarettes) when introducing new products. Consequently, the firm may not be able to create enough awareness before the product is launched and therefore the coefficient of innovation of a diffusion pattern may be smaller than when television advertising is possible [9]. In that case, one may consider product sampling as a substitute to advertising.

In summary, the major contribution of this article has been to provide a parsimonious analytical method to determine the optimal levels of product sampling using a diffusion modeling approach. As noted by Preveger [22], "Sampling has become a much more sophisticated operation than one might have imagined a few years ago." Given this observation, we feel that the results obtained provide useful insights about the nature of impact of product sampling on the diffusion of a new product and also help product managers in developing effective marketing strategies for new product development.

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Appendix: Derivation of the Upper Bounds for the Size of Product Sampling

$F_0 (= N_0/m)$

We consider the general case (target sampling for a repeat purchase product) in which the firm maximizes the net present value given by

$$\pi = \int_0^{\infty} e^{-rt} (p - c) \left(\frac{dN(t)}{dt} + wN(t) \right) dt - (h + c)N_0 \quad (\text{A.1})$$

where

$$\frac{dN}{dt} = \left(a + \frac{b}{m} N(t) \right) (m - N(t))$$

$$N(t = 0) = N_0 \quad (\text{A.2})$$

and

$$a = \alpha + \beta \log \left(1 + \frac{N_0}{m} \right) \quad (\text{A.3})$$

Clearly, the other cases can be obtained from the above formulation as follows:

Neutral sampling for a durable product:	set $w = 0, \beta = 0$
Target sampling for a durable product:	set $w = 0$
Neutral sampling for a repeat purchase product:	set $\beta = 0$.

The first-order condition for optimality for the formulation given in (A.1)–(A.3) is obtained by differentiating (A.1) with respect with N_0 and is given by:

$$c + h = \int_0^{\infty} e^{-rt} (p - c) \left(\frac{\partial}{\partial N_0} \left(\frac{dN}{dt} \right) + w \frac{\partial N}{\partial N_0} \right) dt \quad (\text{A.4})$$

Performing the differentiations and simplifying, we get

$$c + h = \int_0^\infty \frac{e^{-\lambda t}(p - c)\lambda e^{-\lambda t}}{(1 + \delta e^{-\lambda t})^3} \left[\frac{-\lambda^2(1 - \delta e^{-\lambda t})}{(a + bF_0)^2} - \frac{\beta\lambda\delta(1 - \delta e^{-\lambda t})}{b(1 + F_0)(a + bF_0)} + \frac{2\beta\delta(1 + \delta e^{-\lambda t})}{b(1 + F_0)} + \frac{w\lambda(1 + \delta e^{-\lambda t})}{(a + bF_0)^2} - \frac{\lambda t\delta(1 - \delta e^{-\lambda t})}{b} \right] dt \quad (A.5)$$

where

$$\lambda = a + b$$

$$\delta = \frac{b(1 - F_0)}{(a + bF_0)}$$

If $\delta > 1$ then

$$b(1 - F_0) > a + bF_0$$

which implies

$$\frac{1}{2} \left(1 - \frac{a}{b} \right) > F_0$$

Or, using expression (D) from Exhibit 1, we get

$$F_0 < F^*$$

where F^* represents $F(t^*)$ without the argument t . Therefore, all the other upper bounds in Table 1 are established since they involve F^* plus a non-negative quantity.

On the other hand, if $\delta < 1$, then it can be shown (by differentiation) that the expression in the square brackets in (A.5) is decreasing in time. Therefore, at $t = 0$, the expression must be positive, otherwise the right hand side of (A.4) is negative and the equality in (A.5) cannot hold. Evaluating the expression within the square brackets at $t = 0$, we get the following:

$$\left[w(1 + \delta) - \lambda(1 - \delta) - \frac{\beta(1 - F_0)(1 - \delta)}{(1 + F_0)} - \frac{2\beta(1 - F_0)(a + bF_0)(1 - \delta)}{\lambda(1 + F_0)} \right] > 0$$

Substituting for δ and λ in the above equation, we get

$$F_0 < F^* + \frac{w}{2b} + \frac{\beta(1 - F_0)}{2b(1 + F_0)^2}$$

$$< F^* + \frac{(w + \beta)}{2b}$$

since

$$\frac{(1 - F_0)}{(1 + F_0)} < 1$$

References

1. Bass, Frank M. A new product growth model for consumer durables. *Management Science* 15:215-227 (January 1969).
2. Blattberg, Robert C. and Neslin, Scott A. *Sales Promotion*, Englewood Cliffs, NJ: Prentice Hall, 1990.
3. Blattberg, Robert C. and Golanty, John. TRACKER: an early test market forecasting and diagnostic model for new product planning. *Journal of Marketing Research* 15:192-202 (May 1978).
4. *Business Week*. A Few Words on Word Processing. (November 22, 1993).
5. Curtis, Gerald. *Applied Numerical Analysis*, Reading, MA: Addison-Wesley, 1980.
6. Donnelley Marketing Inc. Survey shows popularity of trade promotion and consumer databases. *Potentials in Marketing*, p. 49 (May 1991).
7. Eng, Paul M. Below bargain basement: software freebies. *Business Week* 88 (November 1993).
8. Givon, Moshe, Mahajan, Vijay and Muller, Eitan. Software piracy: estimation of lost sales and the impact on software diffusion. *Journal of Marketing* (in press).
9. Horsky, Dan and Simon, Leonard S. Advertising and the diffusion of new products. *Marketing Science* 2:1-17 (Winter 1983).
10. Jain, Dipak and Rao, Ram. Effect of price on the demand for durables: modeling, estimation and findings. *Journal of Business Economics and Statistics*, 8(2):163-170 (1990).
11. Jones, Morgan J. and Ritz, Christopher J. Incorporating distribution into new product diffusion models. *International Journal of Research in Marketing* 8:91-112 (1991).
12. Kotler, Philip. *Marketing Management*, 7th edition. Englewood Cliffs, NJ: Prentice Hall, 1990.
13. Lazarus, George. Marlboro packs giveaway punch. *Chicago Tribune*, August 25, Section 3, p. 4, (1989).
14. Lilien, Gary L., Rao, Ambar G. and Kalish, Shlomo. Bayesian estimation and control of detailing effort in a repeat purchase diffusion environment. *Management Science* 27:493-506 (May 1981).
15. Mahajan, Vijay, Muller, Eitan and Bass, Frank M. New product diffusion models in marketing: a review and directions for research. *Journal of Marketing* 54:1-26 (January 1990).
16. Mahajan, Vijay and Peterson, Robert A. *Models for Innovation Diffusion*. Beverly Hills, CA: Sage, 1985.
17. Mahajan, Vijay, Muller, Eitan and Srivastava, Rajendra K. Determination of adopter categories by using innovation diffusion models. *Journal of Marketing Research* 27:37-50 (February 1990).
18. Mahajan, Vijay, Wind, Jerry and Sharma, Subhash. An approach to repeat purchase diffusion models. *AMA Proceedings*, Series 49, Patrick Murphy et al. (ed.). Chicago: American Marketing Association, 442-446 (1983).
19. Marks, Lawrence J. and Kamins, Michael A. The use of product sampling and advertising: effects of sequence of exposure and degree of advertising claim exaggeration on consumers' belief strength, belief confidence, and attitudes. *Journal of Marketing Research* 25:266-281 (August 1988).
20. McKenna, Regis. Marketing is everything. *Harvard Business Review* 69:65-79 (January-February 1991).
21. Perkins, G. Anne. Free samples: how many is best. *Harvard Business Review* 72:8-9 (January-February 1994).
22. Preveger, Robert. *Marketing* (U.K.) (November 1989).
23. Priore, Lisa Del. Breaking brand loyalty. *Marketing Communications* 11:40-43 (October 1986).
24. Rogers, Everett M. *Diffusion of Innovations*. 3rd ed. New York: The Free Press, 1983.
25. Schultz, Don E. and Robinson, William A. *Sales Promotion Management*, Chicago, IL: Crain Books, 1982.
26. Silk, Alvin J. and Urban, Glen L. Pre-test market evaluation of new packaged goods: a model and measurement methodology. *Journal of Marketing Research* 15:171-191 (May 1978).