COMMENTS

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Mahajan and Muller have written a valuable paper that analyzes a particular class of advertising policies under the assumption of an S-shaped response function. As we shall discuss, their results highlight the need for new empirical research to answer crucial questions about what constitutes a pulse and the extent to which S-shaped responses exist.

Sasieni (1971), in a brilliant but hard-to-read paper, characterizes optimal advertising policies for a broad class of dynamic response models, including that of Mahajan and Muller. (Mahajan and Muller study awareness whereas Sasieni considers sales, but the mathematics does not care.) As Mahajan and Muller note, Sasieni has shown that an S-shaped response function can, without loss of generality, be replaced by an entirely concave one, if we admit policies in which the advertising rate alternates rapidly between zero and a particular value. Mahajan and Muller denote this value, $\bar{u}$, and illustrate the geometry in Figure 1. By adjusting the proportion of time spent at zero and at $\bar{u}$, a linear response function can be created between the origin and a tangent point where the S-shaped function is concave. Mahajan and Muller call such alternating policies by the apt name "chattering."

Given an unconstrained budget and a concave response function with stationary parameters, Sasieni shows that the optimal (maximal profit) advertising policy will involve constant (even) spending over time, except possibly for a start-up period that may be needed to bring sales up or down to the rate they will have at the optimal even advertising. Thus, the optimal policy always calls for even advertising, where "even" includes the possibility of chattering when the response function is S-shaped and the optimal rate falls in $(0, \bar{u})$.

Sasieni (1975) has subsequently shown in a private communication that the same type of policy is optimal when the advertising budget is constrained. He does this by introducing a Lagrange multiplier for the budget constraint. The result is reasonably intuitive, since the Lagrange multiplier recreates an unconstrained problem. In the limited budget case, there is a reasonable possibility of chattering because the budget constraint may force the advertising rate into the range $(0, \bar{u})$.

Thus, Sasieni has established that even spending is best, where "even" may require rapid alternation between zero and the (even) high level $\bar{u}$. What then are the open questions? I can immediately think of three:

(1) Are there any response models for which pulsing (other than chattering) would be optimal? Obviously, these would have to have some form different from those analyzed by Sasieni and Mahajan and Muller.

(2) Sasieni works with a differential equation in continuous time, as do Mahajan and Muller. Implicit is the assumption that time can be divided as finely as desired. But will the response functions of either paper stay the same as smaller and smaller time intervals are examined? Implicitly, the measurement interval for the continuous model should be quite small. Does this mean a month, a week, an hour, a minute or what? These are tough empirical questions.

(3) As the Sasieni and Mahajan-Muller papers both observe, you cannot, in practice, carry chattering to its mathematical limit of instantaneous switching. Suppose you follow even spending but alternate at some practical rate. How much will this degrade the average performance from Sasieni's instantaneous ideal?

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Of these questions, Mahajan and Muller address only the last. Question (1) on different models has just started to receive attention in the literature. Jones (1983) presents a theoretical study that includes a model that can create hysteresis. H. Simon (1983) has constructed a model that does not fit Sasieni's form, has estimated its parameters from the sales and advertising history of a particular product, and has determined that the optimal policy involves pulsing. Question (2) on subdividing time is an empirical one that, to the best of my knowledge, nobody has addressed. I suspect it is much more difficult than the issue of whether there are any S-shaped response functions to begin with, a continuing controversy in some quarters. (Note that, although Mahajan and Muller discuss S-shaped curves in connection with Zielske's data, their calibration is silent about whether response is actually S-shaped, since they estimate just one point on the curve and assume that $\bar{u}$ = exactly one of Zielske's exposures/week.)

Mahajan and Muller study (3), the effect on performance of varying the frequency of alternating between 0 and $\bar{u}$. Instantaneous chattering is the true optimum by Sasieni's results and so comes out as the upper bound in their numerical examples. However, Mahajan and Muller's calculations suggest that a fairly coarse alternation may suffice. Four times per year achieves 95% of the optimal performance on the Zielske-based example under certain assumptions about the degree of S-shape.

Terminology is rapidly becoming a problem in discussing these phenomena. Part of the reason is the issue of subdividing time and advertising. To take Zielske's experiment as an example, the unit of advertising is a piece of direct mail. He sends it once a week for 13 weeks to one group and once every four weeks for a year to another. He describes the former as a 13-week pulse and the latter as even advertising over the year. Mahajan and Muller call Zielske's even advertising 13 pulses. While there is some reasonableness to this, the schedule is at the physical limit of even advertising with a constrained budget of 13 insertions. In this connection, we note that none of the frequencies in Mahajan and Muller's Figure 4 would have been physically realizable in Zielske's experiment except $k = 1$ and 13, the actual values used. All others require fractional exposures of some sort. Since all advertising is inherently discrete in a fine enough time frame, we need to develop better, probably empirically based, definitions as to what is "even" and what is "pulsed."

Mahajan and Muller's contribution consists of providing a way to calculate the inefficiency of frequencies different from the instantaneous switching ideal. Such a calculation presupposes an S-shaped response function that is known and valid for some underlying unit time period and a budget constraint that forces the advertising into the lower $(0, \bar{u})$ range of the curve.

References (not included in paper)

Jones, P. C. (1983), "Analysis of a Dynamic Duopoly Model of Advertising," *Mathematics of Operations Research,* 8 (February), 122–34.

Sasieni, M. W. (1975), "A Note on Optimal Advertising," private communication.

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I am flattered to see that the data from our 1959 experiment are still being analyzed. And I am impressed with the mathematical elegance of Mahajan and Muller's analysis. It would be presumptuous of me to comment on the technical aspects of the paper. However, some comments on our more recent work in this area may be of interest.

In recent years, we have been using tracking study data to estimate the response curves for various forms of response and various brands. The response forms we have examined include:

(a) Brand Awareness
(b) Advertising Awareness
(c) Brand Image
(d) Brand Preference

To date, we have developed 125 individual curves for 40 brands.

These studies were not controlled experiments. The observed results are a composite of the influence of advertising and the influence of the other market environment factors that were present for the brand. The tracking study data were used to establish what happened as the campaign ran. By using the standard industry formulas, the frequency distribution of exposures delivered by the campaign could be computed.

These two data sets were used to estimate the probability of response according to the number of exposures an individual received. This was done by using simultaneous equations to relate tracking study results at various points in time to the frequency distribution of exposures the schedule had delivered at corresponding points in time.

Virtually all of the curves we computed for awareness were concave—incremental exposures produced successively smaller increases in response. However, they varied widely from brand to brand in how steeply they went up as exposures increased, the level of response that was achieved, and the point at which they began to level off. Sometimes, the maximum response was reached after very few exposures—sometimes response was still increasing even after many exposures—and sometimes there was no increase at all over the prior level.

We define response as the absolute increase that occurred from the level that existed prior to the campaign. Therefore, as would be expected, the curves generally went up more rapidly for new products than for established products. However, the awareness curves were concave for both types of products.

Our conclusions at this point are:

(a) Response curves can be useful for planning.
(b) However, assuming that all brands and all market situations will have the same response curve can be dangerous.

The influence of different forms of scheduling on response is determined by:

(a) The nature of the response curve for the brand.
(b) The decay rate for the response form.

In the market place, response curves for awareness are almost always concave, incremental exposures producing successively smaller increases in awareness.

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REPLY

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REFLECTIONS ON ADVERTISING PULSING POLICIES FOR GENERATING AWARENESS FOR NEW PRODUCTS

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We are indebted to John Little and Hugh Zielske for their valuable comments on our paper. Some of the comments merit further clarification and our response to these comments is provided below.

Little has very appropriately raised the following three important questions:

1. Are there any response models for which pulsing (other than chattering) would be optimal?

2. Since the model formulation assumes continuous time, will the response function stay the same as smaller and smaller (a month, a week, an hour or a minute?) time intervals are examined?

3. Since in practice one cannot chatter, how much does the firm lose by following a practical pulsing policy?

As correctly pointed out by Little, our paper does deal with the third question. We wish that we could provide an answer to his second question. As acknowledged by him, it is a tough empirical question. This question, although very relevant, is beyond the scope of what we have presented in our paper. We hope that some future study will shed some light on this question.

Unless we are misunderstanding Little's first question, we believe that our work does respond to it. In fact, the analytical results presented in §3 and further dissected in the last section of our paper indicate that pulsing is optimal *only* under an S-shaped response function. It is true that the theoretical optimal policy in the S-shaped response function is the chattering policy. However, as demonstrated in §4 (see the simulation results), the optimal practical policy is the pulsing policy. Consider, also, the case when there is a small cost associated with each pulse over and above the regular advertising costs. That is, some kind of transaction cost is involved in each pulse (the cost of the chattering policy then is infinite). The optimal policy then also would be pulsing and what we show in the paper amounts to the fact that even a very small transaction cost would result in an optimal pulsing policy with very few pulses.

We agree with Little's observation that terminology has become a problem in discussing the type of phenomena we are dealing with. In fact, we have attempted to discern some of the confusion associated with the definitional problems in the first footnote of our paper. We hope that we have not added to this confusion by defining $\bar{u}$ as the "threshold" level of advertising. We do realize that dictionaries generally define threshold as "the intensity below which a mental or physical stimulus cannot be perceived and can produce no response." We, however, because of its relevance to our discussion, defined the threshold level as the lowest level of advertising that is still efficient.

In his comment, Zielske notes that virtually all the curves that he and his colleagues have estimated were found to have diminishing marginal returns (i.e., they were concave functions). We do not dispute such observations. In fact, as discussed in the last section of our paper, after reviewing several empirical studies on the shape of the advertising response function, Simon and Arndt (1980) arrived at similar conclusions. We have speculated in the last section of our paper that empirical studies based on aggregated

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advertising levels and responses cannot discover an S-shaped function since the effect of pulsing is to linearize the convex part of the response function. So far as our analyses based on Zielske's data are concerned, the results indicate that for these data, the chattering policy dominates the even policy. By Proposition 3 in our paper this can happen *only* under an S-shaped advertising response function. Thus, we conjecture that the implicit advertising response function in Zielske's famous data is S-shaped.

References

Simon, J. L. and J. Arndt (1980), "The Shape of the Advertising Response Function," *Journal of Advertising Research,* 20 (August), 11–28.