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Majority choice and the objective function of the firm under uncertainty

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A set of production vectors is said to fulfill the "current spanning" condition if any firm's current production vector is spanned by the rest of the market. Our major theorem shows that, given current spanning, a production plan may be found which will always win the approval of a majority of the firm's initial shareholders. Furthermore, these shareholders are shown to be the only ones concerned with the firm's production plans. Finally, we show that an equilibrium exists in which all firms choose plans approved by a majority of their shareholders.

1. Introduction

■ Shareholder decisions in large organizations are made by majority rule, and economic theory has traditionally supported this method of decisionmaking by establishing that there exists a unique, value-maximizing set of investments for the firm which will be the unanimous choice of all shareholders (see, for example, Arrow (1964)). However, since the publication of papers by Jensen and Long (1972) and Stiglitz (1972), it has been known that value maximization may be neither a clearly defined firm objective nor one preferred by a majority (or indeed any) of the firm's shareholders. Subsequent papers by Ekern and Wilson (E-W) (1974), Leland (1974), and Radner (1974) showed that shareholders might be unanimous as to preferences over marginal changes in firm production plans as long as these were made in a limited subspace of all possible plans. Specifically, shareholder unanimity holds if any new production plan does "not alter the state-distributions of returns available in the economy" (E-W, 1974, p. 175).

This condition, which we call the E-W spanning condition, is extremely restrictive. To see this, consider the following example. Suppose there are

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three states and three firms. Assume that current state-dependent return vectors are:

Firm 1: (3,3,3)

Firm 2: (1,2,2)

Firm 3: (2,1,1).

The E-W spanning condition requires that any change in production must be such that the last two components of the new vector of returns are equal. The restrictiveness of the E-W condition is such that it is obvious that in practical situations unanimity will rarely exist; see for example Satterthwaite (1977).

In this paper we introduce a different spanning condition, which we call *current spanning*. This condition requires that each firm's current production vector be spanned by the rest of the market. In the above example, current spanning is guaranteed since the sum of the return vectors of firms 2 and 3 equals the return vector of firm 1. However, the existence of E-W spanning depends on the production technology. If firm 1 adds ϵ to its inputs and the resulting return vector becomes $(3, 3 + \lambda, 3 + \lambda)$, then the E-W spanning condition holds. If, on the other hand, firm 1 adds ϵ to its input and the resulting return vector becomes $(3, 3 + \lambda, 3 + \delta)$, where $\delta \neq \lambda$, then the E-W spanning does not hold, since the new return vector cannot be written as a linear combination of the original return vectors. Note that in both cases current spanning holds, since it does not depend on the production technology.

As we shall show, the current spanning condition guarantees the existence of a production plan which will always win the approval of a majority of the firm's shareholders. Thus, even in incomplete markets, where the E-W spanning condition does not hold, the firm's objective may be established by a majority vote. In the above example current spanning holds; thus, with no restrictions on the production technologies (as required by the E-W spanning condition), majority voting may be used to establish a firm production plan. Moreover, shareholders will have no incentive to misrepresent their preferences in this voting procedure. The literature thus far has dealt exclusively with unanimous shareholder decisions.[1] Clearly, majority rule has a much wider range of application.

The structure of the paper is as follows: We examine the problem which faces a firm in a typical period of an n-period model. The firm must decide how much of its current production income to distribute as dividends and how much to retain for the purchase of firm inputs in the next period. We examine a situation where individuals have made optimal portfolio decisions, given current market prices and firm plans. We then ask about the individual's preferences over retained earnings. Given current spanning, initial shareholders are shown to have single-peaked preferences over the firm's retained earnings, and furthermore, these shareholders will be the only ones concerned with the firm's production plans. In this setting, no individual shareholder will find it advantageous to misrepresent his preferences. Thus, each shareholder will vote sin-

[1] An exception is Gevers (1974). Gevers makes two claims: First, a majority choice equilibrium may not exist. As we show, plausible sufficient conditions exist to invalidate this statement. Second, Gevers correctly states that a majority choice equilibrium may not be Pareto optimal.

cerely, and majority rule will yield a single (and stable) outcome. Finally, we show that an equilibrium exists in which all firms choose plans approved by the majority of their shareholders.

2. The model

■ We suppose there to be only one physical good, whose price will always be one. The assumption about the number of goods is made for simplicity only; as we show in Section 7, the generalization of the model to many goods and many periods is straightforward.

There are I individuals (consumers) and J firms. There are two periods; where the need exists, "today" will be subscripted by 0, and states tomorrow will be indicated by m, $m = 1, \ldots, M$. Thus, for example, the consumption of consumer i will be denoted by $x_i = (x_{i0}, x_{i1}, \ldots, x_{iM})$, where the first component of the vector denotes today's consumption and the other components denote consumption tomorrow if state m obtains. The securities of each of the J firms are traded today (tomorrow the firms liquidate, and there is consequently no trade in their securities). The vector of security prices will be denoted by $p = (p_b, p_1, \ldots, p_J)$, where $p_1, \ldots, p_J$ are the prices of shares of firms $1, \ldots, J$, respectively, and p_b is the price of a riskless bond.

□ **Firms.** Each of the J firms enters period 0 with an endowment of bonds $\bar{b}_j$. As mentioned in the introduction, our two periods are to be thought of as two "typical" periods in an n-period model; thus the endowment of bonds should be thought of as the earnings from last period retained to the current period in the form of bonds. This endowment is now used to purchase inputs z_{j0} for the purpose of producing outputs $y_{j0}(z_{j0})$ in period 0. Since the price of the physical good is one, $y_{j0}(z_{j0})$ is the total production income for firm j in period 0.

In addition to producing in period 0, firm j pays dividends from its production income and retains the remainder of its earnings (if any) in the form of bonds which are redeemed in the next period.[2] If the price of the riskless bond is p_b, and if the firm buys b_j (nominal value) of these riskless bonds, the total amount spent on riskless bonds by firm j will be $p_b b_j$; it is this quantity which we shall call the retained earnings of firm j in period 0.

Production tomorrow is assumed to be stochastic; having retained earnings in the amount of $p_b b_j$ in period 0, firm j will produce $y_{jm}(z_{jm})$ tomorrow if state m occurs, where $z_{jm} = b_j$. In the multigood model, the firm will maximize its production income in state m, subject to the constraint that the costs of the inputs will not exceed the amount b_j. (The function y_{jm} will be assumed to be real-valued, nonnegative, bounded, and concave.)

Firm j may pay dividends to its shareholders in both period 0 and in the first period. There is, however, a difference between dividends paid in period 0 and those paid in the first period: The former are paid to initial shareholders (those individuals i for whom $\bar{f}_{ij} > 0$), while the latter are paid to those whose shareholdings in firm j at the end of period 0 are positive (those i for whom $f_{ij} > 0$). The dividends paid by firm j in the present are $y_{j0}(z_{j0}) - p_b b_j$. In the

[2] Although the introduction of bonds might seem an unnecessary complication, it makes the generalization to many goods a straightforward one.

first period, dividends will depend on the earnings retained by the firm in period 0. If the firm has purchased inputs z_{jm} in state m out of retained earnings, dividends in state m will be $y_{jm}(z_{jm})$.

It is convenient to place one budget constraint on the firm. The reason for doing so will become apparent in the proof of the existence of equilibrium in Section 6. We require that the firm's retained earnings come out of production income in period 0.[3] Thus we require

$$y_{j0}(z_{j0}) - p_b b_j \geq 0. \tag{1}$$

□ **Consumers.** Each consumer receives an endowment of bonds $\bar{b}_i$ in period zero. As in the case of the firm (since we are considering two periods in an n-period model), $\bar{b}_i$ should be thought of as consumer i's debt (if $\bar{b}_i < 0$) from having sold bonds in the "previous" period. If $\bar{b}_i > 0$, it represents repayment of debt from bonds purchased in the "previous" period. In addition, each consumer receives an endowment of the commodity in both period zero and every state of the first period. For consumer i, denote this endowment by $\bar{\omega}_{ih}$, $h = 0, 1, \ldots, M$. We shall assume that all endowments are strictly positive. In addition to his endowment, consumer i will be assumed to possess an initial portfolio of shares at the beginning of the first period. Denoting this portfolio by $\bar{f}_i = (\bar{f}_{i1}, \ldots, \bar{f}_{iJ})$, we shall assume:

$$\bar{f}_{ij} \geq 0, \qquad \text{for every } i \text{ and every } j, \tag{2}$$

$$\sum_i \bar{f}_{ij} = 1, \qquad \text{for every } j. \tag{3}$$

Given firm retained earnings, $p_b b_j$, for each j, consumer i chooses a consumption vector $x_i = (x_{i0}, x_{i1}, \ldots, x_{iM})$, a share portfolio $f_i = (f_{i1}, \ldots, f_{iJ})$, and a nominal amount of bonds b_i. Budget constraints for consumer i are given by

$$x_{i0} \leq \sum_j p_j(\bar{f}_{ij} - f_{ij}) + \sum_j \bar{f}_{ij}(y_{j0} - p_b b_j) + \bar{\omega}_{i0} + \bar{b}_i - p_b b_i \tag{4}$$

$$x_{im} \leq \sum_j f_{ij} y_{jm} + b_i + \bar{\omega}_{im}. \tag{5}$$

Note that we have placed no positivity constraint on consumer i's purchases of new shares. Thus we allow f_{ij} to be either positive or negative; $f_{ij} < 0$ denotes "short sales" by consumer i of firm j's shares.

3. Consumer maximization and implicit prices[4]

■ Each consumer i maximizes a utility function defined on his consumption vector x_i. We shall assume that this function $U_i(x_i)$ is twice continuously differentiable, strictly concave, and increasing in each argument. Furthermore, we assume that

$$\partial U_i/\partial x_{ih} \to \infty \text{ as } x_{ih} \to 0, \text{ for all } h = 0, 1, \ldots, M. \tag{6}$$

[3] Equation (1) follows the usual logic of a corporate society: Shareholders in a firm need not be required to pay for inputs purchased by the firm in excess of its production income. If markets were complete, it might be that all shareholders would be willing to pay for such inputs; since we are primarily concerned with incomplete markets, however, it seems more logical to eliminate this possibility.

[4] The analysis of this section owes much to Baron (1978).

Now assume that, given prices $p = (p_b, p_1, \ldots, p_J)$ and firm retained earnings, consumer i maximizes U_i at (x_i^*, f_i^*, b_i^*). Then strict equality must exist in the budget constraint equations, and we may thus write U_i as a function of the securities portfolio,

$$U_i(x_i^*) = \bar{U}_i(f_i^*, b_i^*). \tag{7}$$

Differentiating to obtain the first-order maximum conditions, we obtain

$$\frac{\partial \bar{U}_i}{\partial f_{ij}} = 0 \Rightarrow p_j = \sum_m \frac{\partial U_i/\partial x_{im}}{\partial U_i/\partial x_{i0}} y_{jm}, \tag{8}$$

where the partial derivatives are evaluated at x_i^*. Letting q_m^i denote the quotient of the partial derivatives so defined yields:

$$p_j = \sum_m q_m^i y_{jm}(z_{jm}), \qquad j = 1, \ldots, J. \tag{9}$$

$$p_b = \sum_m q_m^i. \tag{10}$$

The q_m^i are known as the individual's implicit prices for state m. See Baron (1978). Note that the q's depend on specific firm production plans.

4. Current spanning and consumer preferences over firm plans

■ In this section we introduce the notion of *current spanning*. As previously noted, our purpose is to explain firm behavior in incomplete markets where the E-W spanning property is not satisfied. While current spanning and E-W spanning are disjoint conditions, the implications of the current-spanning condition are less radical than those of E-W spanning. Our result is less radical as well; we shall show that instead of shareholder unanimity, there exists majority agreement among shareholders on production plans.

Current spanning is satisfied when any firm's present vector of returns is spanned by the other firms' current return vectors. Thus, letting $y_h = (y_{h1}, \ldots, y_{hm})$ denote firm h's vector of returns, we have

$$y_j = \sum_{h \neq j} \alpha_{hj} y_h, \qquad \text{for all } j. \tag{11}$$

On the other hand, the E–W spanning condition states that any new production plan obtained by a small change in the current production plan is spanned by current return vectors. Clearly, barring second-order effects, the condition can be stated as

$$y_j' = \sum_h \beta_{hj} y_h.$$

That is, the marginal production is spanned by current return vectors.

We now proceed to give two examples which show that the two conditions are independent. They will also illustrate the intuition behind current spanning.

Example 1: E-W spanning and not current spanning. Let there be three states and three firms. Assume that at current bond sales, b_1, b_2, b_3, the return vectors are

$$y_1(b_1) = (3,3,3)$$
$$y_2(b_2) = (1,2,2)$$
$$y_3(b_3) = (1,2,2).$$

The E-W condition requires that if $y_1(b_1 + \Delta b_1)$ is the new production vector when Δb_1 is added to inputs, then

$$y_1(b_1 + \Delta b_1) = (3 + \lambda_1 \Delta b_1, 3 + \lambda_2 \Delta b_1, 3 + \lambda_2 \Delta \mathrm{b}_1).$$

Thus, as long as changes in production are equal in the last two coordinates, the market satisfies E-W spanning. Note that in this example current spanning does not exist, since y_1 cannot be written as a linear combination of y_2 and y_3.

Example 2: current spanning and not E-W spanning. Let there be 3 firms and 3 states. Assume that at current bond sales, b_1, b_2, b_3, the return vectors are

$$y_1(b_1) = (3,3,3)$$
$$y_2(b_2) = (1,2,2)$$
$$y_3(b_3) = (2,1,1).$$

Clearly, since $y_1 = y_2 + y_3$, current spanning exists. On the other hand, suppose $Y_1(b_1 + \Delta b_1) = (3, 3 + \lambda\Delta b_1, 3 + \delta\Delta b_1)$. Then $y_1(b_1 + \Delta b_1)$ cannot be spanned by y_1, y_2, y_3 as long as $\lambda \neq \delta$.

Note that the current spanning condition requires linear dependence of the vectors of returns. However, linear dependence is not a sufficient condition; what is required is that condition (11) hold for every j. The following example shows that this is stronger than linear dependence: Suppose we add a fourth firm to the present example:

$$y_4(b_4) = (1,1,2).$$

Then, while the set (y_1,y_2,y_3,y_4) is linearly dependent, y_4 cannot be written as a linear combination of the first three. Finally, note that current spanning does not imply completeness; for example, if a fourth zero component is added to the vector of returns of the above example, the market will still satisfy current spanning but will be incomplete.

Theorem 1: Suppose that firm j, $1 \leq j \leq J$, chooses b_j^* and that (x_i^*, f_i^*, b_i^*) maximizes U_i, $1 \leq i \leq I$. Then, if current spanning holds, an individual i who holds shares of the firm at period zero (i.e., $\bar{f}_{ij} > 0$) will prefer that firm j choose retained earnings such that b_j maximizes net present value of the firm:

$$p_j - p_b b_j. \tag{12}$$

Furthermore, the maximization of (12) is equivalent to the maximization of

$$\sum_m q_m^i y_{jm}(z_{jm}(b_j)) - p_b b_j \tag{13}$$

with the implicit prices $\{q_m^i\}$ taken to be constant.

Note: Two remarks are appropriate at this point. First, we do not assume that the implicit prices $\{q_m^i\}$ are constant. Rather, we shall prove that personalized maximization of the net present value when current spanning holds is equivalent to maximizing (13) *as if* the $\{q^i_m\}$ were constant. Second, note that (13) is derived by substituting equation (9) into equation (12). Since implicit prices may vary among individuals and since both (12) and (13) depend on implicit prices, individuals will, in general, *not* agree on the value-maximizing level of retained earnings b_j.

Proof: Since p_b is positive, maximization with respect to the retained earnings is equivalent to maximization with respect to the bonds b_j. Differentiating U_i yields the first-order conditions

$$\frac{\partial U_i}{\partial b_j} = \frac{\partial U_i}{\partial x_{i0}} \frac{\partial x_{i0}}{\partial b_j} + \sum_m \frac{\partial U_i}{\partial x_{im}} \frac{\partial x_{im}}{\partial b_j} = 0. \tag{14}$$

The current spanning assumption allows consumer i to achieve any desired consumption vector with $f_{ij} = 0$, since firms $h \neq j$ span firm j's production vector. Thus we may write the budget constraints (4) and (5) as

$$x_{i0} = \sum_j \bar{f}_{ij}(p_j - p_b b_j + y_{j0}) - \sum_{h \neq j} f_{ih} p_h + \bar{\omega}_{i0} + \bar{b}_i - p_b b_i \tag{15}$$

$$x_{im} = \sum_{h \neq j} f_{ih} y_{hm} + b_i + \bar{\omega}_{im}. \tag{16}$$

Therefore, (14) becomes

$$\frac{\partial U_i}{\partial b_j} = \frac{\partial U_i}{\partial x_0} \bar{f}_{ij}\left(\frac{\partial p_j}{\partial b_j} - p_b\right) = 0. \tag{17}$$

Thus, if $\bar{f}_{ij} = 0$, (17) always holds. If $\bar{f}_{ij} > 0$, (17) is the necessary condition for the maximization of (12).

To establish the second part of the theorem, recall equation (9). We now have

$$\frac{\partial p_j}{\partial b_j} - p_b = \frac{\partial}{\partial b_j}\left(\sum_m q^i_m y_{jm}\right) - p_b = \sum_m q^i_m \frac{dy_{jm}}{db_j} + \sum_m y_{jm} \frac{\partial q^i_m}{\partial b_j} - p_b. \tag{18}$$

Recalling (15) and (16), we have

$$\begin{aligned} \frac{\partial p_j}{\partial b_j} - p_b = {} & \sum_m q^i_m \frac{dy_{jm}}{db_j} \\ & + \sum_m y_{jm}\left[\frac{\bar{f}_{ij}}{U_0^2}\left(\frac{\partial p_j}{\partial b_j} - p_b\right)\left(U_0 \frac{\partial U_m}{\partial x_{i0}} - U_m \frac{\partial U_0}{\partial x_{i0}}\right)\right] - p_b, \end{aligned} \tag{19}$$

where U_0 and U_m denote the appropriate partial derivatives of U_i. Therefore,

$$\frac{\partial p_j}{\partial b_j} - p_b = \frac{1}{c}\left[\sum_m q^i_m \frac{dy_{jm}}{db_j} - p_b\right], \tag{20}$$

where

$$c = 1 - \sum_m y_{jm}\left(U_0 \frac{\partial U_m}{\partial x_{i0}} - U_m \frac{\partial U_0}{\partial x_{i0}}\right)\frac{\bar{f}_{ij}}{U_0^2}.$$

By checking the second-order condition, it is clear that equating (20) to zero will yield a sufficient as well as necessary condition for maximization of (13) or equivalently (12). This completes the proof.[5]

Note 1: It might be asked how the behavioral assumptions underlying equations (15) and (16) can be integrated into a general equilibrium context, since in these equations we have explicitly assumed that consumer i reasons as if $f_{ij} = 0$. In Section 6 we shall show that these assumptions are, in fact, consistent with general equilibrium. As we shall show, in equilibrium all of the following hold simultaneously:

(1) Each individual chooses an optimal portfolio given firm actions. With current spanning, this portfolio is nonunique and thus leads to the choice outlined in Theorem 1.

(2) Were a vote to be held, each individual would vote as specified in Theorem 1. In fact, the equilibrium choices of the firms reflect the majority choice of initial shareholders, and there will be no further changes in retained earnings.

(3) In equilibrium some individual i holds some of firm j (this follows from the equilibrium conditions), and this individual—even though he might prefer different retained earnings for firm j—will not find it advantageous to change his portfolio, since no firm will change its production/retained earnings decision (point 2 above).

Note 2: Theorem 1 is central to the paper. It might be appropriate to ask whether the results may be obtained under other assumptions. If we assume that each individual predicts the change in market prices following a change in firm policy on the basis of his present implicit prices, then Theorem 1 still holds. There is a certain amount of "rationality" in such an assumption: In equilibrium, equation (9) always holds, and each individual i, knowing that in previous periods his implicit prices have always "predicted" market prices, might well assume that the same holds for the next period.

The implicit prices q_m^i may be viewed as consumer i's present value for one unit of production in state m. Thus, Theorem 1 states that each initial shareholder prefers that firms in which he holds shares maximize the (personalized) net present value of production. It is intuitively obvious that shareholders may fail to agree about firm objectives simply because their implicit prices differ. This conflict was long ago resolved by Arrow (1964). If the returns of the securities span the set of states, the market is called *complete*. It is well known that in this case all implicit prices will be equal. We may therefore define a set of state prices as the common implicit prices of consumers, and—provided these state prices are viewed as given—net present value is an appropriate objective function, and unanimity will follow.

That markets are not, in general, complete makes it difficult for us to claim that shareholders will, in most cases, be unanimous as to firm choice.

[5] In the proof we should have dealt with the case where $c = 0$. However, this case may be solved trivially by one of the following methods: A suitable change in the initial condition $\bar{f}_{ij}$ and $\bar{\omega}_{i0}$ will guarantee that $c \neq 0$ without changing the individual's optimum. Alternatively, assume that for at least one state m, the second derivative $U_{m0} = 0$. This is true, for example, for all states m if U is a von Neumann-Morgenstern utility function. Then $c = 0$ becomes a cardinal property and may be changed by a suitable transformation of the utility function.

5. Majority choice

■ In this section we show that, given current spanning, there always exists a choice of retained earnings for each firm which will be supported by a majority of the initial shareholders.

Theorem 2: Suppose that firm j chooses its retained earnings $p_b b_j^*$ and that (x_i^*, f_i^*, b_i^*) maximizes U_i for $i = 1, \ldots, I$. Then, if current spanning holds, there exists $p_b \hat{b}_j$ which is preferred by a majority of initial shareholders (weighted by their initial shareholdings) to any other choice of retained earnings $p_b b_j$. Moreover, in this setting, no individual shareholder will find it advantageous to misrepresent his preferences, and thus each shareholder will vote sincerely.

Note: Before we prove the theorem, several points should be noted. First, $\hat{b}_j$ may be equal to b_j^*. If this is the case for every firm j and if, furthermore, demands and supplies of all commodities and shares are equal, we have an equilibrium. In the next section we shall show that such an equilibrium (which we call a majority choice production equilibrium) exists. Second, it follows trivially from the previous discussion and from our proof of Theorem 2 that if markets are complete, the criterion of majority choice is equivalent to the Arrow-Debreu criterion. Third, although we assume that the production functions are concave, this is a sufficient condition which is not necessary. In the one good case discussed here, it is easy to construct an example of an S-shaped production function which will yield the same result.

Proof: By Theorem 1, each shareholder maximizes the expression (13):

$$\sum_m q_m^i y_{jm}(b_j) - p_b b_j$$

with respect to b_j (or $p_b b_j$). Since (13) is a nonnegative combination of concave functions combined with a linear function, it is concave. It now follows from our assumption that there exists a finite solution to the maximization problem of each consumer, that his preferences over the retained earnings of the firm are single-peaked.[6]

It is well known that with single-peaked preferences and an odd number of voters, majority rule is a well-behaved procedure in the sense that it yields a single alternative as the outcome. This single alternative has the property that it cannot be defeated, in a pair-wise comparison, by any other alternative. (For basic references, see Black (1948, 1956) or Arrow (1963).)

If each production function has one state m in which it is strictly concave, then consumers' preferences are strictly concave in retained earnings, and each consumer has a single most-preferred value of retained earnings. In this case, each voter reports his most-preferred alternative and the group's choice is the median of all these reported most-favored choices. This procedure is clearly nonmanipulable, as the only way a voter can change the outcome is in a direction which is lower than his preference.

If the production function is not strictly concave in any state, it is possible

[6] The existence of a finite solution is guaranteed by our assumption that the production functions are bounded (Section 2).

that consumers' preferences are not strictly concave, and that the most-preferred level of retained earnings is not a single point, but rather lies in an interval. Black (1956) calls this "single peaked with a plateau" and shows that the majority choice procedure is essentially the same for this case as for the strictly concave case. In this case—as well as in other cases involving an even number of voters or an irrational number of shares, for example—a tie-breaking procedure may be needed. Furthermore, an appropriate choice of tie-breaker leads to a unique outcome. *Q.E.D.*

The majority choice (with possible tie-breaker) which we have invoked in Theorem 2 consists essentially of choosing the median of the expressed preferences of initial shareholders. Since the median of an array of numbers is a continuous function of the elements of the array, we have the following corollary:

Corollary: The outcome of the voting procedure presented in Theorem 2 is a continuous function of the individual reported (and sincere) preferences over the retained earnings $p_0 b_j$.

6. The existence of a majority choice production equilibrium

■ The aim of this section is to establish the existence of an equilibrium in which every firm chooses retained earnings which have the approval of the majority of its shareholders. The proof follows traditional equilibrium proofs, and we shall give details sparingly.

Definition: p^*, (x_i^*, f_i^*, b_i^*), (z_j^*, b_j^*) is an equilibrium if

(E1) $\sum_i \bar{\omega}_{i0} + \sum_j y_{j0}(z_{j0}^*) - \sum_i x_{i0}^* - \sum_j z_{j0}^* = 0;$

(E2) $\sum_i \bar{\omega}_{im} + \sum_j y_{jm}(z_{jm}^*) - \sum_i x_{im}^* - \sum_j z_{jm}^* = 0;$

(E3) $\sum_i f_{ij}^* = 1, \qquad \text{for all } j;$

(E4) $\sum_i b_i^* + \sum_j b_j^* = 0;$

(E5) consumers maximize the utility of consumption subject to the budget constraints (4) and (5); and

(E6) firms choose retained earnings which would not be defeated by a majority of their initial shareholders, were these shareholders to maximize the firm's (personalized) net present value of production viewing their implicit prices as constant.

Note: If current spanning holds, then by Theorem 1 consumers do, in fact, maximize the net present value of production viewing their implicit prices as constant. As we noted after the proof of Theorem 1, however, there may be other circumstances under which the implicit prices could be viewed as constant. The above formulation allows for the more general case.

Lemma: (E3) and (E4) ⇒ (E1) and (E2).

Proof: By assumption, consumers are insatiable. We may, therefore, assume equality in the budget constraint equations (4) and (5) for each consumer i.

Summing these equations, and assuming that there was equilibrium in the previous period gives the desired result.[7] *Q.E.D.*

By the above lemma we need consider only one market, namely the securities market, in our search for equilibrium.

We now define a bounded economy as follows: Let A_0 be a bound on feasible consumption and input vectors

$$A_0 = \sum_i \bar{\omega}_{i0}.$$

Similarly, we may bound consumption in state m by A_m, where

$$A_m = \sum_i \bar{\omega}_{im} + \sum_j y_{jm}(A_0).$$

Now note that

$$-\hat{A}_0 = -(A_0 + \sum_i \bar{f}_{ij}p_j + \sum_i \bar{b}_i + \sum_j \bar{b}_j) < f_{ij}p_j$$
$$< A_0 + \sum_i \bar{f}_{ij}p_j + \sum_i \bar{b}_i + \sum_j \bar{b}_j = \hat{A}_0.$$

To see this, note that $f_{ij}p_j$ is the total payment given (received, in the case of short sales) when securities of firm j are bought (shorted) today. Since A_0 is the bound on all wealth in the first period, at equilibrium no individual can spend (receive) more than this sum from purchases (sales) of any security.

Note that the same bound applies to b_i.

Now define

$$F_{ij} = f_{ij}p_j,$$

and write the set of *permissible vectors* for consumer i

$$\hat{E}_i(p,(z_j)) = \{(x_i,F_i,b_i) \big| \quad |F_{ij}| \leqq \hat{A}_0, \quad |b_i| \leqq \hat{A}_0$$
$$0 \leqq x_{i0} \leqq A_0, \qquad 0 \leqq x_{im} \leqq A_m,$$
$$x_{i0} \leqq \sum_j \bar{f}_{ij}p_j - \sum_j F_{ij} + \sum_j \bar{f}_{ij}(y_j - p_b b_j) + \bar{b}_i - p_b b_i + \bar{\omega}_{i0},$$
$$x_{im} \leqq \sum_j \frac{F_{ij}}{p_j} y_{jm} + b_i + \bar{\omega}_{im}\}.$$

We are now in a position to define the preference functions of individuals and firms. For individual i define

$D_i(p,(z_j)) = D_i(p, z_1, \ldots, z_J) = \{(x_i, F_i, b_i) \big| , x_i$ maximizes U_i over all permissible vectors, and (x_i,F_i,b_i) is permissible$\}$.

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[7] Implicit in all our arguments are the following two conditions:

$$\sum_i \bar{b}_i + \sum_j \bar{b}_j = 0$$
$$\bar{\omega}_{i0} + \bar{b}_i > 0, \qquad \text{for all } i.$$

The first of these conditions may be thought of as an equilibrium condition from the previous period. The second condition we shall need to guarantee is that individual i has the possibility of consuming $x_{i0} > 0$. In a multiperiod model, both of these conditions would automatically be fulfilled.

For firm j define

$D_j(p,(z_h),(x_i)) = D_j(p, z_1, \ldots, z_J, x_1, \ldots, x_I) = \{\hat{z}_j | \hat{z}_j$ is the majority decision given $x_1, \ldots, x_I\}$.

It is easily established that E_i is convex, and that D_i and D_j are continuous, convex-valued set functions, given our assumption that the utility functions U_i have continuous derivatives. We may thus use the Kakutani fixed point theorem and standard equilibrium proof techniques (see, for example, Arrow and Hahn (1971) or Debreu (1959)) to establish the following theorem, which we give here without proof:

Theorem 3: There exists a majority choice production equilibrium.

7. Extensions and conclusions

■ Our model can be extended to more than two periods as follows. In a rational-expectations framework, such as that of Radner (1972), the firm has to announce its plan for all time-event pairs, given prices and consumer plans. By a procedure similar to that used in the proof of Theorem 1, it may be shown that in a framework of a sequence of one-period models, preferences about firm plans in any period t are held only by shareholders who purchased shares of the firm in the previous period $t - 1$. But since this is exactly the case discussed in our paper, it follows that our procedure can remain essentially unchanged for the multiperiod case.[8]

To extend our results to the multicommodity case, note that if the firm buys b_j (nominal value) of riskless bonds in period 0, it will maximize the first-period production income, subject to the constraint that the costs of the inputs may not exceed b_j. Thus, the firm's income I_{jm} in period 1 is:

$$I_{jm}(b_j) = \text{Max } \{y_{jm}(z_{jm}^1, \ldots, z_{jm}^k)\},$$

subject to

$$\sum_{h=1}^{k} p_m^h z_{jm}^h \leqq b_j,$$

where $z_{jm}^1, \ldots, z_{jm}^k$ are the k inputs in state m, and where $(p_m^1, \ldots, p_m^k)$ is the commodity price vector associated by all consumers with state m. Thus, we assume, as does Radner (1972), that "traders have common expectations, . . . [i.e.,] they associate the same (future) prices to the same events."

Since y_{jm} is concave in the inputs, it is well known that $I_{jm}(b_j)$ will be concave in its argument b_j. Initial shareholders will have preferences on the retained earnings which are both concave and single-peaked in this case, since they maximize expression (13) where $I_{jm}(b_j)$ is substituted for y_{jm}.

To conclude, we have shown that under the current spanning assumption, shareholders have single-peaked preferences over the firm's retained earnings. Their expressed preferences over the retained earnings will be their true preferences, and majority rule will yield a single (and stable) outcome. The initial shareholders will be the only individuals to have strong preferences (i.e., be nonindifferent) over the retained earnings, and each one of them prefers that

[8] For a discussion of the multiperiod rational expectations model with securities markets, see Benninga (1979).

firms in which he holds shares maximize the net present value of production. Moreover, an equilibrium exists in which all firms choose plans approved by a majority of their shareholders.

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