# Satiation and Cross Promotion:
# Selling and Swapping Users in Mobile Games

Michael Haenlein

Barak Libai

Eitan Muller

November 2021

Michael Haenlein is Professor in the Marketing Department at ESCP Business School and holds the Chair in Responsible Research in Marketing at the University of Liverpool Management School (haenlein@escp.eu)

Barak Libai is Professor of Marketing at the Arison School of Business, Interdisciplinary Center, Herzliya, Israel, libai@idc.ac.il.

Eitan Muller is Research Professor of Marketing, Stern School of Business, New York University, New York, NY, 10012; and Professor of Marketing, Arison School of Business, Interdisciplinary Center, Herzliya, Israel, emuller@stern.nyu.edu.

The authors wish to thank Eyal Biyalogorsky, Ron Shachar and seminar participants at HEC Paris, Tilburg University, University of Zurich, Ben-Gurion University, IDC, Trinity College Dublin, WHU, and the University of Maryland for their helpful comments and suggestions. This work has been supported by the Israeli Science Foundation.

**Satiation and Cross Promotion:**
**Selling and Swapping Users in Mobile Games**


## Abstract

One of the main challenges to the mobile game industry is an alarming level of satiation, that is, a decline in user engagement and consequently in ad viewing, spending, and retention. Satiation lowers users' CLV to the extent that makes acquisition from the likes of Facebook and Google untenable, driving the game publishers to cross-promote, that is, sell and swap users among themselves. We model this cross-promotion as first, a ***screening mechanism*** in that the fact of playing a game indicates specific preferences that might be suitable for an exchange with likewise games, and second, as a ***resetting mechanism*** that allows the swapped users to reset their engagement in the new game, thus making the swap or sell beneficial to both buyer and seller. We show that there exists an optimal level of satiation to a game, and with this level, we show the conditions under which the game publisher cross promotes, and when it does, what are the conditions for selling rather than swapping. We extend the analysis to the case in which advertising costs and conversion rates are related, explain why they might be negatively correlated, and show that our main results still hold.

# 1. INTRODUCTION

Given the social shift from focusing on experiences rather than goods, consumers increasingly turn to online hedonic experiences such as music streaming, mobile games, and YouTube videos (Morgan 2019). These experiences often suffer from the effect of satiation, where the repeated consumption of the same hedonic experience is likely to produce a decline in usage and liking (Galak and Redden 2018). This decline in usage leads to a decline in usage-related revenues such as advertising or in-app purchases, leading to low customer lifetime value (CLV) that often makes acquisition costs in places such as Facebook or Google untenable. Given a declining value of the customer over time, there can be a point at which the user's value can be higher for a third party, in particular in a new experience where s/he is not yet satiated, leading to the willingness of the firm to "sell" the customer.

While a customer cannot easily be sold like any other asset, firms can incentivize proactive churn by giving existing customers access to the offers of competing firms. Such proactive churn management is prevalent in the mobile games industry, where firms often sell advertising slots to competitors who may use them to advertise their products (Han et al., 2016; Appel et al., 2020). The focal firm engages in such behavior, knowing it may result in churn for some existing customers. We use the mobile game industry nomenclature to label this type of proactive churn, ***cross-promotion***[1].

We examined mobile game industry publications, research reports, followed presentations in industry conventions, and interviewed executives in global firms, both gaming publishers and ad networks. In doing so, we observed a unique combination of motivations and

[1] There are two common usages to the term “cross-promotion”: The act of advertising at a competitor’s game (Lee et. al 2020), and the resultant transfer of a user from one competitor to the other via swapping or selling. We use both, as the context unambiguously reveals the meaning of the term.

abilities that makes the mobile gaming industry a potential projection into new models of customer management in an increasingly data-driven digital world. In particular, we see the following four phenomena in these markets:

- **First, an ecosystem characterized by intense cash-flow pressure:** Casual mobile games start for free, CLV is low, and customer acquisition costs are high. To survive this cash-flow pressure, games publishers have developed a profound ability to manage users on an individual level that is way ahead of most other markets. This ability draws on an ecosystem that includes a) external sources of customer acquisition such as Facebook and Google that enable smart customer acquisition based on customer knowledge, b) data-science based ad networks that mediate between advertisers and app publishers, and c) app publishers that can follow time-sensitive user behavior in-depth and continuously communicate with them.
- **Second, the ubiquity of cross-promotion:** We observe the prevalence of cross-promoting customers to increase profits, particularly in casual and hyper-casual games whose monetization is built around advertising. As reported by Luz (2019): "*Currently, the majority of those advertising in-game are other games*." Indeed, a senior manager for a sizeable hyper-casual game publisher admitted in an interview that "*to a large extent, much of our efforts are about buying and selling customers*." An executive in a major global ad network estimated that most games he deals with, publish click-through advertising to other competing games, which often leads to churn. Market reports which assess customers' churn from ads from different sources advise publishers to analyze the effect of churn on *CLV* to decide on their advertising policy (Lerner 2019).
- **Third, the selling and buying of users within the same publisher portfolio:** The pricing mechanism that allows for selling and buying users is also used when selling occurs within games of the same publisher. The reason is that games are managed by brand managers who are reluctant to give away a customer to another game of the same publisher unless compensated by a reasonable price. Our interviews suggest that while app publishers may be aware of the possibility of churn via ads from other apps, they do not necessarily analyze the total effect on customer value, and some are reluctant to use cross-promotion.

- **Fourth, the emergence of blacklisting:** A common practice in the mobile gaming industry is blacklisting, i.e., the process of preventing the promotion of specific apps or apps belonging to specific categories (Kim 2020, Digital Limbo 2019). Because cross promotion may involve swapping customers, by blacklisting the firm affects not only the type of customers it transfers, but also the ones it can expect to get. There is a discussion and some criticism among industry observers on the utility of blacklisting, and the question of its contribution is still open.

Still, publishers do not always rely on cross-promotion, and customer acquisition sources such as Facebook and Google still account for a large part of new customer acquisitions. This motivates our investigation here: We examine via a formal model the market conditions that will lead to cross-promotion in which brands sell their customer to other brands. We consider a game publisher faced with the decision to acquire a customer. The game publisher can acquire customers from *outside market* sources such as Facebook and Google, considering the customer acquisition cost (*CAC*) vs. the expected *CLV* of the customer. What cross-promotion brings into the picture are the possibilities in an *inside market*, i.e., cooperating with another game. Such cooperation can occur by placing a click-through ad in another game and then paying per installed user or swapping customers and "paying" with one of their customers.

We model this cross-promotion as first, a ***screening mechanism*** in that the fact of playing a game indicates specific preferences that are suitable for an exchange with likewise games, and second, as a ***resetting mechanism*** that allows the swapped users to reset their engagement in the new game, thus making the swap or sell beneficial to both buyer and seller. We assume that customer satiation is endogenous: firms can invest in reducing the level of satiation, given all buyers and sellers' alternatives in both inside and outside markets.

The equilibrium created from this complex echo system enables us to understand the conditions under which selling and swapping customers emerge. We show that it is the quality of the inside market, regardless of the quality of the outside market, that determines if the game publisher cross promotes. We also explain why the likelihood of observing cross-promotion decreases in retention and increases in satiation, and why given cross-promotion is chosen, the likelihood of observing swapping decreases with gross profits and retention and increases with the cost of designing a game. Our analysis helps explain why blacklisting that prevents the cross-promotion of specific apps paradoxically tends to increase the likelihood of cross-promotion and not reduce it. We extend the analysis to the case in which advertising costs and conversion rates are related, explain why they might be negatively correlated, and show that our main results still hold.

Our framework and findings can be of interest also beyond the (noteworthy) market for mobile games. An example is the market for personalized content recommendations in online news outlets (Song, Sahoo, and Ofek 2018). Like mobile games, online news outlets suffer from satiation of users and are actively redirecting customers to other media organizations via recommendation platforms. The technological capability of sophisticated intermediaries such as Taboola and Outbrain to conduct data-based analysis and increase profitability to all sides is pivotal in this market, as in mobile games. In an interview with the CEO of one of these two firms, he noted the extreme difficulties he had when the firm was a budding startup to convince commercial websites to send customers away to competitors, and the need to convince about satiation and the benefit of transferring customers. As selling customers may become relevant wherever customer satiation and the ability to manage customers intersect, an in-depth analysis of these intriguing markets can be of much interest.

## 2. BACKGROUND

Our research relates to research streams on customer satiation, customer profitability, recommendation mechanisms, and mobile app monetization.

**Customer Satiation**: The fact that people grow tired of repeatedly experienced stimuli reduces consumption even in the presence of satisfaction with the product and therefore has robust implications for managing customers. The level of satiation and the resulting customer variety-seeking behavior has been shown to impact a firm's marketing management, such as the types of products carried, the monetization mechanisms used, and the optimal design and pricing strategies applied (Caro and Martínez-de-Albéniz 2012; Appel et al. 2020; Sajeesh and Raju 2010). A rich behavioral literature has emerged in recent years, exploring the role of satiation in consumer markets. Much of these efforts focus on identifying ways to mitigate hedonic satiation (Galak and Redden 2018; Lasaleta and Redden 2018; Sevilla et al. 2016). For example, firms may want to create breaks in consumption such as TV commercials, change the consumption rate, or encourage consumers to anticipate future variety (Nelson et al., 2009; Galak et al., 2013; Sevilla et al. 2016). Cross-promotion is a different approach to mitigating the effects of satiation rooted in the principles of customer management: Instead of changing the behavior of a given customer, the customer is transferred, and his/her low engagement is reset to a higher level with the new hedonic experience.

**Customer Profitability**: Our work is consistent with the view of customers as assets to be managed, which is at the base of the CRM literature (Gupta and Lehmann 2003; Rust, Lemon, and Zeithaml 2004). Work in this area has examined issues such as the importance of managing a customer portfolio (Johnson and Selnes 2004), the interaction with other assets of the firm (Fang, Palmatier and Grewal 2011), and the use of customer acquisition, retention, and development to manage the customer asset (Bolton, Lemon, and Verhoef 2004; Lewis 2006). Within the CRM literature, it is accepted that the expected profitability of customers should be considered in the decision of investment and targeting of customer acquisition (Peters, Verhoef and Krafft 2015) and that the mode of acquisition (e.g., discounts, word of mouth) can affect the consequent CLV (Lewis 2006; Villanueva, Yoo, and Hanssens 2008). We add to this impressive literature the idea of the firm's ability to profit from the asset by proactive churn, which should also be considered in the resource allocation for customer acquisition.

**Recommendation Mechanisms**: Cross-promotion is, in many cases, initiated through an advertisement for a competing product or service. This advertisement is *de-facto* a recommendation of the firm to try another product. Firms can recommend products to customers in various ways. The best known is *cross selling* (Knott et al. 2002; Li et al. 2011; Prins and Verhoef 2007; Schmitz et al. 2014). In the digital world, cross-selling can be done via recommendation systems that suggest products based on similarity to the consumption of other customers (Oestreicher-Singer et al. 2013); and by enabling sellers to send links to other sellers in social commerce networks (Stephen and Toubia 2010, Dellarocas et al. 2013). One issue that distinguishes cross-promotion from work on other recommendation mechanisms is the need to directly consider the loss of customer lifetime value when the customer is transferred.

**Monetization of Mobile Apps**: Finally, our work joins an emerging literature that deals with the monetization of mobile apps. Much attention has been given to the app publisher's tradeoff off, balancing revenues between the free version and the paid version of the app. Recent efforts in this domain have focused on the issues of pricing and design (Cao, Chintagunta and Li 2021), satiation (Appel et al. 2020), network effects (Shi, Zhang, and Srinivasan 2019), and longer-term customer retention (Ascarza, Netzer, and Runge 2020). Our work looks at the challenges of app monetization from a different angle - the terms under which app publishers will use cross-promotion, further monetizing customers on the one hand and creating an inside market customer acquisition alternative on the other.

## 3. CROSS PROMOTION IN MOBILE GAMES

**Mobile Game Environment**: Mobile games are the world's most popular form of gaming, growing fast, and are by far the largest app category, with estimations of more than $40b in the first half of 2021 and with more than half of all app revenues in App Store and Google Play combined app revenues (SensorTower 2021). It is estimated that more than two and half billion people around the globe, heterogeneous in age and with balanced gender representation, are playing mobile games (Silver 2020). The mobile game industry is a vibrant echo system with continually emerging new business models regarding profit creation (Choi et al. 2020).

One can divide mobile games into three broad types: C*ore games* (e.g., Clash of Clans) are often targeted at specific niches and generally require players to invest significant time to learn. As a result, core games have the highest engagement among games. *Casual games* (e.g., Candy Crush) have more of a mass-market appeal. They typically have more straightforward game mechanics and rules and can be picked up quickly. *Hyper casual games* (e.g., 2048) are even simpler to learn. They are instantly playable with very little learning time, require scant attention, and have intuitive mechanics that remain consistent throughout gameplay (Karnes 2020). Hyper-casual games are the most significant game type regarding the number of downloads (but not revenues) among mobile games (AppFlyer 2019).

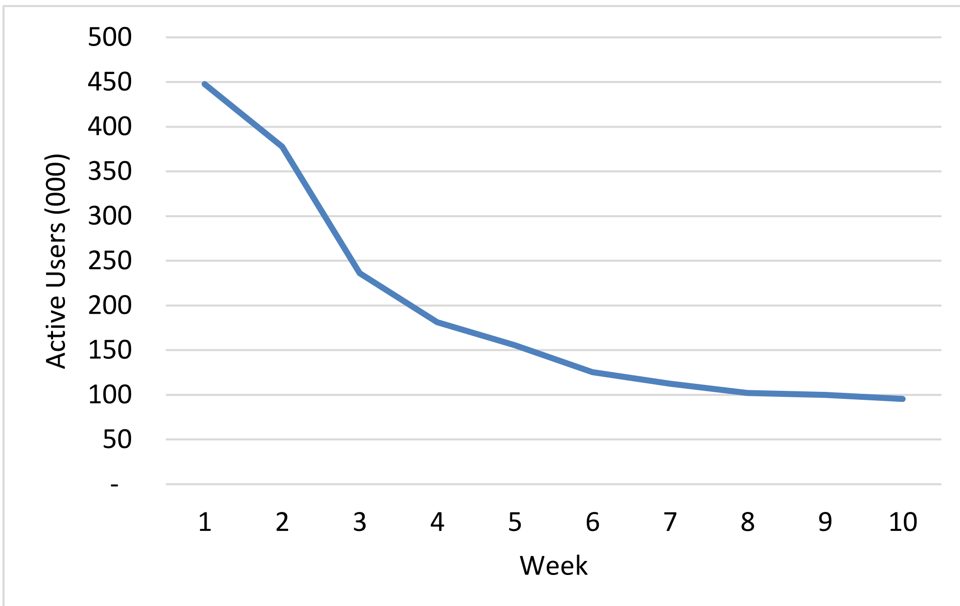
In terms of monetization, more than 90% of mobile games start for free (free-to-play), relying mainly on two mechanisms of monetization: in-app purchases and advertising (Appel et al. 2020). In-app purchases are the source for most revenues to date, particularly for core and casual games. However, the contribution of advertising has grown much in recent years, and hyper-casual games are primarily monetized by advertising (Frid 2019).

**Motivating Evidence of Cross Promotion**: To motivate our investigation and illustrate the effects we discuss, we present some model-free evidence from an established publisher of multiple mobile games, several of which reached the Top 100 in the major app stores in recent years. We obtained data on adoption, retention, and usage of nine consecutive games, where each game was introduced one week after the launch of the previous one. This example represents a case of *internal cross-promotion*, where a customer is transferred to another brand within the publisher's brand portfolio. Game publishers have created a portfolio of products to ensure that the satiated customers of one brand will become the new customers of another of their brands (Popescu 2020), and large game publishers such as Disney use this strategy in mobile games (Wong 2016).

The similarity to external cross-promotion comes because brand managers increasingly act independently and choose whether to use internal cross-promotion or acquire from an outside source. The cross-promotion process is outsourced to an advertising network that uses its data science capabilities to find the right candidate, similar to finding an outside partner (e.g., IronSource 2021). In such cases, while the game publisher that owns the portfolio profits from both the seller and the buyer, the buyer and seller can maximize their independent profits in a way that resembles the analysis that is our focus here.

**The Fast Decline of Active Users**: Figure 1 depicts the number of active users of one of these casual games (Game 3). Consistent with industry reports on casual games, we see a fast decline where only about a quarter of players are active in two months. This fast decline could be explained by the difficulty in retaining users who did not pay (Datta, Foubert, and Van Heerde 2015) and the satiation that characterizes most mobile games (Han et al. 2016; Hui 2017).

**Figure 1: Active Users of a Casual Mobile Game (000)***




* To preserve confidentiality, the numbers of active users are multiplied by a constant $1 \leq \theta \leq 1.3$

**The Extent of Cross Promotion**: Table 1 shows the size of cross-promotion across the nine games. We see, for example, that 10% of the users of Game 10 come from Game 9, an additional 4.5% came from Game 8, and that a total of 22.7% of all users of Game 10 come from cross-promotion. Looking only at games with at least four previous games feeding into it (Games 5 and above), we see that cross-promotion effect size systematically exceeds 20%, with an average of 28.6% for Games 5 to 10. Therefore, more than a quarter of each game's user base stems from cross-promotion, which shows the overall considerable size of the phenomenon.

**Table 1: Cross Promotion Size across Nine Causal Mobile Game***

| Receiving Game | | Feeding Game | | | | | | | | | | Total Cross Promoted |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **Game 10** | **Game 9** | **Game 8** | **Game 7** | **Game 6** | **Game 5** | **Game 4** | **Game 3** | **Game 2** | **Game 1** | |
| | **Game 10** | - | 10.0% | 4.5% | 2.1% | 1.3% | 1.0% | 0.8% | 0.5% | 1.8% | 0.7% | **22.7%** |
| | **Game 9** | | - | 14.9% | 4.8% | 3.5% | 1.1% | 0.9% | 0.8% | 3.2% | 1.4% | **30.6%** |
| | **Game 8** | | | - | 15.0% | 8.3% | 3.3% | 2.0% | 1.8% | 2.8% | 3.6% | **36.8%** |
| | **Game 7** | | | | - | 10.0% | 4.8% | 2.0% | 1.7% | 2.4% | 4.1% | **25.0%** |
| | **Game 6** | | | | | - | 12.7% | 5.8% | 3.6% | 3.4% | 4.6% | **30.1%** |
| | **Game 5** | | | | | | - | 14.0% | 4.6% | 4.2% | 3.5% | **26.3%** |
| | **Game 4** | | | | | | | - | 7.4% | 5.1% | 2.9% | **15.4%** |
| | **Game 3** | | | | | | | | - | 15.9% | 2.6% | **18.5%** |
| | **Game 2** | | | | | | | | | - | 0.9% | **0.9%** |
| | **Game 1** | | | | | | | | | | - | **0.0%** |

* For example, 10% of the users of Game 10 came from Game 9, an additional 4.5% came from Game 8

**Retention, Satiation, and Cross Promotion**: Table 2 shows an exemplary dataset for one of the games (Game 3 of Figure 1). Note from Table 2 that we know the total number of sessions and average session length per week for each game. This allows us to determine the total activity (number of sessions times average session length) and, by dividing this measure by the total number of active users, we compute the average activity per active user in the last column of the table. This last column clearly shows a decline in activity per user over time that is consistent with satiation.

To show the relationship between satiation and cross-promotion, we run a log-linear regression for each game where the activity per user at time *t* is given by $\text{Activity per user}_t = \text{Baseline activity} \cdot \delta^{\,t}$, where $\delta$ is a parameter that captures the extent of satiation. Similarly, we can measure retention for each game: We look at the evolution of the number of total active users for each game, and following a log-linear regression, similar to the one we use for satiation, we compute the average retention rate for each app. In Table 3, we see the cross-promotion size and the estimated satiation and average retention for each of the nine games.

**Table 2: Exemplary Dataset (Game 3)***

| Week | Total Acquisitions (000) | Total Active (000) | Number of Sessions (000) | Session Length (minutes) | Activity per User (minutes) |
|---|---|---|---|---|---|
| 1 | 447 | 448 | 1,653 | 7.2 | 26.7 |
| 2 | 185 | 378 | 1,681 | 6.1 | 27.2 |
| 3 | 56 | 236 | 900 | 5.4 | 20.7 |
| 4 | 33 | 181 | 635 | 5.1 | 18.0 |
| 5 | 25 | 155 | 510 | 5.3 | 17.4 |
| 6 | 16 | 126 | 409 | 5.2 | 17.0 |
| 7 | 13 | 112 | 353 | 5.4 | 16.8 |
| 8 | 10 | 102 | 318 | 5.2 | 16.2 |
| 9 | 13 | 100 | 308 | 5.2 | 16.1 |
| 10 | 13 | 95 | 305 | 5.7 | 18.1 |

* To preserve confidentiality, the numbers of active users are multiplied by a constant $1 \leq \theta \leq 1.3$

We see that the correlation between cross-promotion size and the satiation parameter is substantial and significant (-0.65). Therefore, the smaller the satiation parameter, that is, the higher the satiation (as low $\delta$ implies high satiation), the higher the size of the cross-promotion effect. We also see that the correlation between cross-promotion size and retention rate is substantial and significant (-70%). Therefore, the smaller the retention rate, the larger the size of the cross-promotion effect.

**Table 3: Cross-promotion and Satiation***

| **Game** | **Cross-promotion** (from Table 1) | **Satiation ($\delta$)** | **Retention** |
|---|---|---|---|
| Game 10 | 23% | 90% | 65% |
| Game 9 | 31% | 94% | 76% |
| Game 8 | 37% | 92% | 72% |
| Game 7 | 25% | 94% | 76% |
| Game 6 | 30% | 92% | 79% |
| Game 5 | 26% | 94% | 79% |
| Game 4 | 14% | 95% | 89% |
| Game 3 | 19% | 95% | 84% |
| Game 2 | 1% | 96% | 90% |
| **Correlation with cross-promotion** | | **-65%** | **-70%** |

* low $\delta$ implies high satiation, thus cross-promotion positively correlates with satiation

Beyond the interest to the specific game publisher, the question is whether the relationship between cross-promotion, satiation, and retention can be generalized. To understand that, one needs to conduct a comprehensive analysis that considers the interests of the buyers and sellers of the inside market and the outside market alternative they have. We will now show that these relationships emerge as a generalization on the market conditions that drive cross-promotion.

## 4. MODELLING CROSS-PROMOTION

We analyze a game publisher (*the buyer*) that considers its options for customer acquisition. We focus on active acquisition via payment, which is the most common form (AppsFlyer 2020). While we do not restrict our analysis to any game type, the market we describe is consistent with the casual and hyper-casual game categories, where cross-promotion has become a ubiquitous option. We describe a case with two market players: a buyer and a seller. Both are game publishers. We assume each game publisher has one customer at the point of analysis and that the buyer wants to acquire an additional customer. The seller is a game publisher willing to transfer such a customer to the buyer if approached and if the price paid is worthwhile. A summary of the parameters used in the analysis is given in Table 4.

**Table 4: Model Parameters***

| Parameter | Description | Comments |
|---|---|---|
| | **Exogenous Parameters** | |
| $\alpha$ | Market quality of the inside market | $0 \leq \alpha \leq 1$ |
| $\beta$ | Market quality of the outside market** | $0 \leq \beta \leq 1$ |
| $r$ | Retention probability per period of current customer | $0 \leq r \leq 1$ |
| $g$ | Gross profit margins per period of current customer | |
| $c$ | Cost parameter of designing a game | |
| *CAC* | Customer acquisition cost from the outside market** | |
| | **Endogenous Constructs/Parameters** | |
| $\delta$ | Decline in gross profit margins per period due to satiation*** | $0 \leq \delta \leq 1$ |
| $\mu$ | Decline in retention probability per period due to satiation*** | $0 \leq \mu \leq 1$ |
| $T$ | Time at which the buyer aims to gain a new customer | $1 \leq T$ |
| *PAC* | Purchase acquisition costs from the inside market | |
| *CLV* | Expected lifetime value of a current customer at time 1 | |
| *RSV* | Expected residual lifetime value of a current customer at time *T* | |

* An additional parameter is the discount rate *d* that plays no role in mobile games, where the average stay is measured in weeks. For completeness' sake, we include it yet disregard it in all sensitivity analyses

** We do not make any assumption on the relative qualities of the inside and outside markets ($\alpha$ and $\beta$). In Section 6, we deal with the case where *CAC* and the quality of the outside market are related

*** Low $\delta$ and $\mu$ imply high satiation

**Satiation, *CLV*, and *RSV***: Customer satiation leads to lower customer engagement with the product over time as users spend less time in the game and will be less exposed to advertising and in-app purchases. Under satiation, customers will be more likely to churn. Thus, satiation can affect the two fundamental components of customer profitability: per period gross margin ($g$) and per period retention probability ($r$). Hence, the values of $g$ and $r$ are only starting values. For example, $g$ is a measure of "fit" to the game, i.e., how much the customer likes the game and is engaged initially, yet this declines over time with satiation. We assume a declining expected profit pattern and retention due to satiation in the following functional form, where $\delta \leq 1$, and $\mu \leq 1$:

(1) $\begin{Bmatrix} g(t) = g \cdot \delta^{t-1} \text{ for } t \geq 1 \\ r(t) = r \cdot \mu^{t-1} \text{ for } t \geq 1 \end{Bmatrix}$

Note that when a customer is transferred, a new satiation process starts. The ***resetting mechanism*** of behavior from a satiated customer to a customer who begins a new satiation process is an essential source of profitability increase that drives cross-promotion. Hence, if the firm's discount rate per period is *d*, the expected profitability over time is as follows in Table 5 (where churn happens at the beginning of each period). Satiation influences two basic profitability measures: The expected CLV of a customer just acquired (*CLV*) and its residual value (*RSV*). The expected CLV of a new customer under satiation is, therefore:

(2) $CLV_{1\to\infty} = \frac{rg}{1+d-\delta\mu r}$

At time T, the remaining lifetime value, which we label the residual lifetime value (RSV), of a customer for the seller is:

(3) $RSV_{T\to\infty} = (\delta\mu)^T \frac{rg}{1+d-\delta\mu r} = (\delta\mu)^T CLV_{1\to\infty}$

Equation 3 shows that *RSV* takes account of the decay driven by satiation the customer experiences up to *T*. Note that this analysis is done at time *T* when the transaction takes place, and thus *RSV* takes as given that the customer is still active, that is, she has been retained so far. In case the game developer would like to find out the residual value at the time it acquires a customer (time 0), then the odds of the customer staying as an active user should be taken into account, and thus the value would be $r^T(\delta\mu)^T CLV_{1\to\infty}$. However, when calculating the transaction value at time T, both buyer and seller already know that the customer is still active, so no conditional probability is needed.

**Table 5: Expected profit of an app with satiation $\delta$ and $\mu$**

| $t = 1$ | $t = 2$ | $t = 3$ | $t = T$ |
|---|---|---|---|
| $g\frac{r}{1+d}$ | $rg\delta\frac{r\mu}{(1+d)^2}$ | $rg\delta^2\frac{r^2\mu^2}{(1+d)^3}$ | $rg\delta^{T-1}\frac{r^{T-1}\mu^{T-1}}{(1+d)^T}$ |

**Optimal Satiation**: The extent to which customers show satiation will depend on the game's characteristics and any possible influence of customer characteristics. For example, games with a unique concept, a more challenging in-game experience, or a competitive character are likely to be more interesting to play over a more extended period, leading to less satiation. If there is no cost in developing and publishing a game with low satiation, then the problem is trivial: the firm will only publish games with high $\delta$ and $\mu$ (implying low satiation) – ideally $\delta = \mu = 1$ – which maximizes the CLV expressed in Equation 2. In the following, we assume that the cost of managing satiation follows a quadratic shape:

(4) $C(\delta\mu) = \frac{1}{2}c(\delta\mu)^2$

The firm is hence faced with optimizing the following profit function:

(5) $\pi = CLV_{1\to\infty} - C(\delta\mu) = \frac{rg}{1+d-\delta\mu r} - \frac{1}{2}c(\delta\mu)^2$

In Web Appendix A, we show that when the firm chooses satiation level subject to a quadratic cost structure, then there is a cost level $\bar{c}$ such that for $c > \bar{c}$ there exists a unique internal solution $\delta^* = \mu^* < 1$ that maximizes profit while for $c \leq \bar{c}$ the firm chooses the boundary condition $\delta^* = \mu^* = 1$. We also show that the optimal level of satiation $\delta^*$ increases in gross profit, cost of design, and underlying retention that is:

(6) $\partial\delta^*/\partial g \geq 0$, and $\partial\delta^*/\partial r \geq 0$

Thus, for the remainder of our analysis, we assume that satiation follows the optimal level as specified in Web Appendix A. Moreover, as the case of $\delta^* = \mu^* = 1$ is hardly of interest, we assume that the cost parameter $c$ is larger than the threshold level, thus $\delta^* = \mu^* < 1$.

We next examine the potential benefit of the buyer and the seller under the three options the buyer has at its disposal: ***acquisition*** (from outside sources such as Google or Facebook), ***purchase*** (from another game publisher in the inside market, which we label the *seller*), or ***swap*** (with another game publisher in the inside market)[2].

**Buyer Option 1 – Acquisition from the Outside Market: Facebook & Google**: Acquisition sources on the outside market are external entities such as Google, Facebook, or Snap, through which a buyer can acquire new users. Google and Facebook alone were estimated to represent close to half of all game advertising investments, and games are considered a significant source of income for these two platforms (Seufert 2019). The strength of these outside market sources comes from the comprehensive view of their users' behavior in out-of-game environments and intelligent targeting algorithms that enable them to reach potential users. When approaching the outside market, the buyer considers two factors: the customer acquisition cost (*CAC*) and the quality of the outside market, which is often measured in the conversion rate.

Customer acquisition cost (*CAC*) is the cost paid to the advertising provider of the outside market. Outside market sources will typically have a price for customer acquisition (which can be fixed or based on auctions) regardless of the specific advertiser, so *CAC* is exogenously determined to the game we analyze. However, CAC can be affected by the expected quality of the specific outside market outlet.

[2] In Web Appendix A we show the conditions under which the market breaks down and no exchange takes place.

The quality of the outside market ($\beta$) measures the ability of the outside market to target advertising to prospects that fit the game well. One can also see this outside market quality as a conversion rate representing the ability to capture high-level customers from the outside market. We assume that the effective expected CLV for the outside market prospect will be the industry level $CLV$ multiplied by $\beta$ with $0 \leq \beta \leq 1$. We assume a homogenous outside market in terms of customer quality, yet we can also assume that customer quality follows a uniform distribution, thus allowing for the possibility of distribution in the quality of the customers, that is $\beta = U(\beta_{min}, \beta_{max})$. The rest of the analysis holds, mutatis mutandis, with $\bar{\beta} = (\beta_{min} + \beta_{max})/2$, replacing $\beta$. Overall, the benefit of the buyer after acquiring at the outside market (and keeping the customer they already have) is:

(7) $Benefit_{B1} = \beta CLV - CAC + RSV$

Thus, for simplicity in the following analysis, CAC is fixed. Section 6 examines a case where CAC is a function of market quality and shows that our basic results still hold.

**Buyer Option 2 – Purchase from the Seller on the Inside Market: A screening mechanism**: Instead of relying on the outside market, the buyer can approach the inside market of other game publishers and pay to get a customer. Mobile games differ from most other apps in their ability to draw on other games in their customer acquisition process. In a sense, the mere fact of playing a game indicates specific preferences and, therefore, serves as a ***screening mechanism*** for other games looking for customer acquisition, most likely with an ad network's help. Given the large number of potential sellers and buyers in the market, and the difficulty efficiently connecting them, advertising networks can mediate among publishers and advertisers. Using large amounts of data on individual users and on the needs, availability, and history of the publisher's customer base, ad networks can use data science to optimize advertising in a way beyond the capabilities of the typical publisher or advertiser (Dogtiev 2020a). When approaching the inside market, the buyer considers two factors: the purchase acquisition cost (*PAC*) and the quality of the inside market.

The purchase acquisition cost is the cost the buyer must pay for a customer that installs the game (we show later that it results from the equilibrium conditions). We use the label *PAC* to differentiate it from the customer acquisition cost on the outside market. Purchase from the seller on the inside market occurs if the buyer prefers purchasing over acquiring on the outside market and the seller prefers selling over swapping. In this case, the buyer pays the seller the purchase acquisition cost *PAC*. From the seller's perspective, *PAC* is equal to the "salvage value" of the customer at time *T*, when the exchange occurs.

Like the quality of the outside market, the quality of the inside market ($\alpha$) reflects the fit of the potential user to the game ($0 \leq \alpha \leq 1$). As before, one can see α as a conversion rate representing the ability to capture high-level customers from the inside market. The question here is how much playing in one game serves as a helpful screening mechanism for another game. The similarity between the games plays a role: If the seller's game is similar to the buyers' in terms of the characteristics and preference of the players, the seller can expect higher quality customers $\alpha$, which will raise the effective *CLV*. Like the case of the outside market quality, we assume a homogenous market in terms of customer quality, yet we can also assume that customer quality follows a uniform distribution, thus allowing for the possibility of distribution in the quality of the customers, that is $\alpha = U(a_{min}, a_{max})$. The rest of the analysis holds, mutatis mutandis, with $\bar{\alpha} = (\alpha_{min} + \alpha_{max})/2$, replacing $\alpha$. The benefit for the buyer after the second option is thus:

(8) $Benefit_{B2} = \alpha CLV - PAC + RSV$

**Buyer Option 3 – Swap with the Seller on the Inside Market: Saving on out-of-pocket costs**: Customer swap also happens among players of the inside market. However, in a swap, the buyer pays the seller not by transferring *PAC* but by providing the seller access to its customers. By doing so, the buyer becomes a seller, and de-facto customers can be "swapped."

Customer swap occurs in practice in two ways: direct or via an app network exchange (Dogtiev 2020b; Rankmyapp 2019). In direct cross-promotion, two apps connect directly and agree to send each other traffic. Large advertising networks may enable brands that work with them to use their database to search for potential partners for free as part of their service. This operation will be associated with some transaction costs of finding the right partner of mutual interest and creating the agreement. An app network exchange is a platform such as Tappx or Tapdaq that mediates among the sides (Salz 2015, Banis 2018). The platform can use its data science market knowledge to offer informed matches to potential game publishers and save them negotiation costs. The platform provides credit to a focal app for allowing other apps to advertise in them, and in return, the focal app can use the credit to advertise itself in other apps. Since the buyer gives away its current customer, the benefit after the deal is:

(9) $Benefit_{B3} = \alpha CLV$

**The Seller Options**: Of course, the market transaction can occur only in a case where both sides benefit. This has two implications: First, it is conceivable that none of the options mentioned above is attractive for the buyer. In this case, the buyer may decide that it is not worth paying for additional customers and, consequently, forgo the decision to gain a new customer. Second, for options 2 and 3, which occur on the inside market, the seller must find the deal attractive. We, therefore, have to consider the seller's benefit as well.

Like the buyer, the seller has three options: Option 1 is to keep the customer and not engage in an exchange with the buyer (which will happen if the buyer approaches the outside market or decides not to invest in acquisition at this stage). Option 2 is to transfer the customer to the buyer on the inside market. In this case, the seller obtains the purchase acquisition cost *PAC* from the buyer. However, to keep the level of market activity and make the comparison to other options valid, the seller also needs to acquire a customer on the outside market in exchange for the customer acquisition cost *CAC*. Option 3 is to swap the customer with the buyer if the buyer is interested in a swap. The benefits of these three options to the seller are as follows:

(10) $Benefit_{S1} = RSV$

(11) $Benefit_{S2} = PAC + \beta \cdot CLV - CAC$

(12) $Benefit_{S3} = \alpha CLV$

## 5. WHEN DO WE EXPECT CROSS-PROMOTION TO OCCUR?

In this section, we analyze the equilibrium and identify the conditions under which cross-promotion occurs, and once it does, we identify the conditions under which either selling or swapping is optimal. In order to find these conditions, we have first to find the equilibrium condition of the purchasing acquisition costs *PAC*, as it has crucial implications for both buyer and seller. While Web Appendix B specifies the exact parameter constraints for each of these conditions, we focus here on purchasing on the inside market that determines its price. For purchasing on the inside market to occur, we need the following four conditions:

1. The buyer prefers purchasing from the inside market over acquiring on the outside market
2. The buyer prefers purchasing over swapping
3. The seller prefers selling over keeping the user
4. The seller prefers selling over swapping

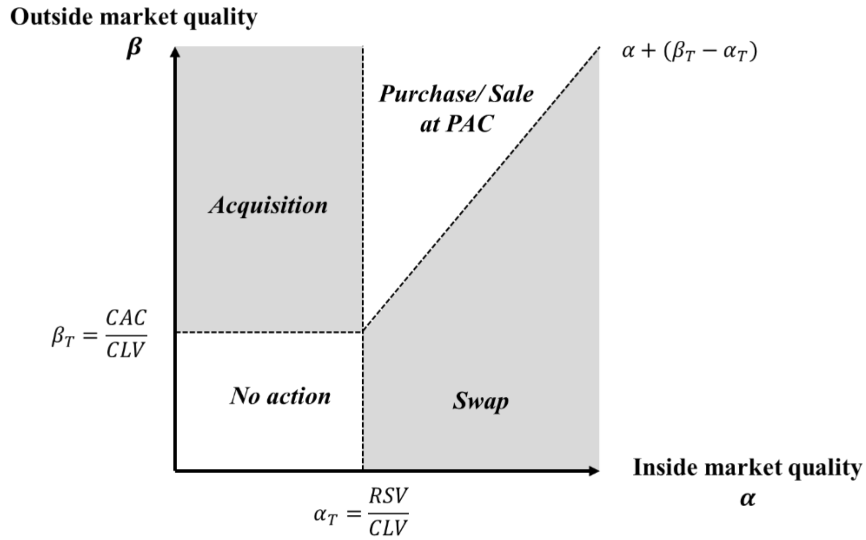
Condition 1 implies the following inequality following Equation 8 and 7 that specifies that the buyer prefers purchasing from the inside market over acquiring on the outside market $\alpha CLV - PAC + RSV \geq \beta \cdot CLV - CAC + RSV$, which is an upper bound on *PAC*. Similarly, condition 4 implies a lower bound on *PAC* following Equations 11 and 12. These two bounds can be satisfied only with the following equation for the Purchasing Acquisition Costs:

(13) $PAC = (\alpha - \beta) \cdot CLV + CAC = \alpha CLV - (\beta CLV - CAC)$

Equation 13 shows that the purchase acquisition cost of a customer reflects the difference between the value the seller supplies to the buyer ($\alpha CLV$) and the value the buyer can get in the outside market alternative ($\beta CLV - CAC$). Hence, the user's salvage value differs from other customer profitability measures such as *CLV* that focus on the customer and its relationship with the firm, in the sense that it takes account of the interplay of two entities – the buyer and the seller. The customer relationship with the firm ($CLV$) is part of the salvage value calculation, but so is the extent the customer fits a prospective buyer ($\alpha$), the quality of the alternative market for the buyer ($\beta$), and the alternative acquisition cost ($CAC$).

**Market Quality and the Decision to Cross-Promote**: Conditions 1 and 4 specify the exact price of the exchange between the seller and the buyer (*PAC*). Similarly, for swapping, we will get $\alpha CLV \geq RSV$. We label the expected market outcomes – acquisition, purchase, swap, and no action – as the *customer buying zones*. Figure 2 presents a graphical demonstration of the different customer buying zones, where the vertical axis represents the outside market quality ($\beta$), and the horizontal axis is the inside market quality ($\alpha$).

**Figure 2: Market Equilibrium Outcomes**




The *inside market quality threshold* is defined by $\alpha_T = RSV/CLV$, i.e., the relative residual value of the current customer, defined as the fraction of residual value out of *CLV*. If inside market quality is high (i.e., above the threshold), the equilibrium outcome is purchase/ sale at *PAC* or swap. The intuition here is that a high-quality inside market indicates that the seller's screening is valuable for the buyer, which makes the acquisition of a screened (vs. random) customer more beneficial. However, if the inside market quality is low (i.e., below the threshold), screening has only limited value, and the choice is between acquiring a random/ non-screened customer on the outside market or no action.

The *outside market quality threshold* is defined by $\beta_T = CAC/CLV$, i.e., the relative customer acquisition cost on the outside market, defined as the fraction of customer acquisition cost from CLV. In the case of low inside market quality where the choice is between acquisition on the outside market and no action, acquisition only occurs when the outside market's quality is sufficiently high. The intuition here is that a buyer would only acquire a customer on the outside market if the expected benefit ($\beta$ *CLV*) is more than the cost of such an acquisition (*CAC*). If this is not fulfilled, the buyer prefers no action over acquisition.

The diagonal line represents a second threshold of outside market quality defined by $\alpha + (\beta_T - \alpha_T) = \alpha - \frac{RSV - CAC}{CLV}$. The line has a fixed slope of 1, and an intercept equal to the difference between the relative customer acquisition cost and relative residual value $(\beta_T - \alpha_T)$. If inside market quality is high, the choice is between purchase/ sale and swap. We observe purchase/ sale if the outside market quality is high (above the threshold) and swap if it is low (below the threshold). The intuition here is that in the case of purchase/ sale, the seller must acquire a new customer on the outside market to replace the sold customer. Hence the outside market quality becomes essential. Since the buyer has to give up *RSV*, and the seller has to acquire a new customer on the outside market for *CAC*, the relevant threshold needs to consider both variables.

> **Proposition 1**: In equilibrium, the game publisher cross promotes when the quality of the inside market is above a threshold, regardless of the quality of the outside market. The latter comes into play in deciding whether to swap if the quality of the outside market is low relative to the inside market or else to purchase from a rival.

**Proof of Proposition 1**: The proof relies on Table 6 that summarizes the results of Figure 2:

**Table 6: Summary of Market Equilibrium Outcomes**

| | | Cross Promotion | |
|---|---|---|---|
| | **Acquisition from the Outside Market** | **Purchase from Seller on the Inside Market** | **Swap with the Seller on the Inside Market** |
| Quality of the inside market ($\alpha$) | $\alpha \leq \alpha_T$ | $\alpha \geq \alpha_T$ | $\alpha \geq \alpha_T$ |
| Quality of the outside market ($\beta$) | $\beta \geq \beta_T$ | $\beta \geq \alpha + (\beta_T - \alpha_T)$ | $\beta \leq \alpha + (\beta_T - \alpha_T)$ |
| Costs to the buyer per one user | $CAC$ | $PAC = (\alpha - \beta) \cdot CLV + CAC$ | $RSV$ |

* $\alpha_T = RSV/CLV$ and $\beta_T = CAC/CLV$

Since $\alpha_T = RSV/CLV$ is independent of the outside market quality $\beta$, it implies that the decision to cross-promote is only concerned with the size of $\alpha$ relative to $\alpha_T$. On the other hand, the decision to purchase on the inside market or to swap depends on both levels of inside and outside market qualities given by the size of $\beta$ relative to $\alpha + (\beta_T - \alpha_T)$.

**Market Factors and the Decision to Cross-Promote**: Next, we consider the outside and inside market quality thresholds that govern switches among the different customer buying zones in Figure 2. For outside market quality, the threshold is the relative acquisition cost of the buyer in the outside market (*CAC*/*CLV*). This relative acquisition cost is a well-established measure across industries to assess customer acquisition (Oba 2017). The threshold for inside market quality is the relative cost of letting go of the customer to the seller (*RSV/CLV*). We use the expressions for *CLV* and *RSV* given in Equations 2 and 3 that results in the following expressions:

(14) $\alpha_T = \frac{RSV}{CLV} = (\delta\mu)^T$

(15) $\beta_T = \frac{CAC}{CLV} = CAC\,\frac{1+d-\delta\mu r}{rg}$

Given Proposition 1, the inside market threshold (equation 14), which is driven by users' satiation, governs the decision to cross-promote[3]. The seller's benefit from selling on the inside market, considering the price paid, is $\alpha CLV$, and in exchange for this benefit, the seller gives up RSV in return. At the time of customer acquisition, the seller does not have an incentive to transfer the customer because satiation has not materialized yet, and the residual value of the customer for the seller is still too high. Over time satiation will become more pronounced, which makes transfer more attractive for the seller. In the absence of satiation ($\delta = \mu = 1$), the inside

[3] As we showed above discussing optimal satiation, there is no difference in the effect in this context between the retention satiation $\delta$ and the usage satiation $\mu$. When we mention "satiation", we refer to both.

threshold becomes one, meaning there is no inside market. Cross-promotion (either through selling or swapping), therefore, only occurs when satiation is present.

We can now also further examine the market factors that affect satiation. We have seen (see Equation 6 and Web Appendix A) that the satiation parameter is higher (and hence satiation lower) if (a) gross profit margins increase, (b) the cost of designing games decreases, and (c) the retention probability increases. Thus, these market factors will also affect the decision to cross-promote. Proposition 2 summarizes the sensitivity of cross-promotion to the market conditions:

> **Proposition 2**: In equilibrium, the likelihood of observing cross-promotion (either selling to or swapping a user from a rival) increases in satiation and decreases in retention[4]. It also decreases in gross profits and increases in the costs of designing a game.

**Proof of Proposition 2:** Figure 2 shows that all equilibrium outcomes correspond to areas in a square defined by $\alpha$ and $\beta$. Since both $\alpha$ and $\beta$ are between 0 and 1, the total size of that square is 1. We can, therefore, interpret the size of each area as a measure of how likely a specific outcome is to occur – see Figure 2.

Given this approach, we define $A$ as the likelihood of observing outside acquisition in equilibrium, $P$ as the likelihood of observing inside purchase/ sale at $PAC$ at equilibrium, $S$ as the likelihood of observing swap at equilibrium, and $CP = P + S$ as the likelihood of observing any form of cross-promotion (either selling or swapping). It is straightforward to calculate the areas of these shapes of Figure 2 to come up with the following (in Appendix C, we provide another scenario in which the line $\alpha + \beta_T - \alpha_T$ cuts the $\alpha = 1$ line above 1, and in this scenario, the sizes of $S$ and $P$ are different):

[4] This result is consistent with the empirical observation we presented in the motivating example.

$$(16)\quad \begin{cases} A = \alpha_T(1-\beta_T) \\ S = \beta_T(1-\alpha_T) + 0.5(1-\alpha_T)^2 \\ P = (1-\alpha_T)(1-\beta_T) - 0.5(1-\alpha_T)^2 \\ CP = 1-\alpha_T \end{cases}$$

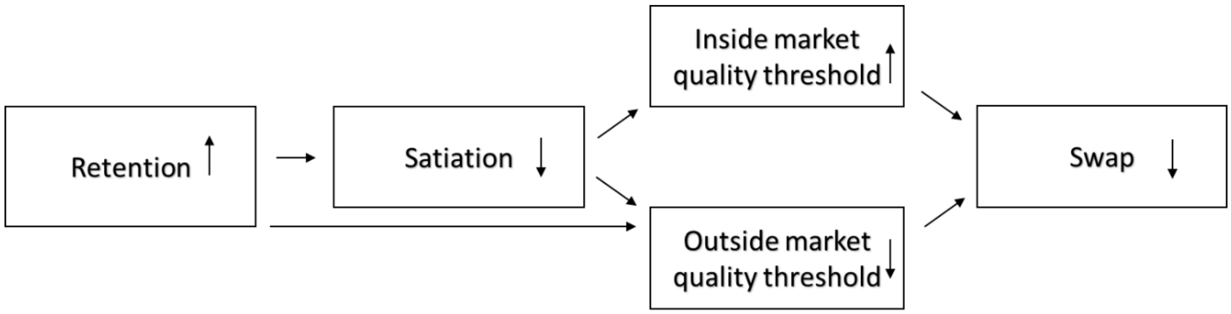
The proof follows chain differentiations of these quantities (see Web appendix C for details).

**Market Factors and the Decision to Sell or Swap**: While Proposition 2 investigated the conditions for cross-promotion (regardless of whether selling or swapping), Proposition 3 delves into the case where the firm decides to cross promotes and then wonders if it should sell or swap. This decision depends on the value of the diagonal line in Figure 2, which separates swapping and selling. An analysis of the market factors that affect the diagonal line yields the following proposition.

> **Proposition 3**: In equilibrium, given that the firm has decided to cross-promote, the likelihood of observing swapping decreases with gross profits and retention and increases with the cost of designing a game.

**Proof of Proposition 3:** The proof follows Equation 16 and a chain differentiation of the terms *S* and *P*, with respect to *g*, *r*, and *c* (see Web Appendix C). For example, to see the effect of retention, observe the following sequence (similarly for other parameters): An increase in retention decreases satiation (increases $\delta$), which in turn increases the threshold of the inside market quality ($\alpha_T$) and decreases the threshold of the outside market quality ($\beta_T$), the latter also decreases directly via the increase in retention (see Equation 13). An increase in the threshold of the inside market quality ($\alpha_T$) decreases the likelihood of swap (since the latter requires the inside market quality to be larger than this threshold), and similarly, a decrease in the outside market quality threshold decreases the likelihood of swap (see Figure 2). Thus, this sequence that began with an increase in retention decreases the likelihood of a swap. This is summarized in Figure 3:

**Figure 3: Retention Leading to Swap (and similarly for selling)**




**The Effect of Blacklisting on the Ubiquity of Cross Promotion**: The emergence of inside markets confronts managers with critical issues less relevant in outside markets. One of them is blacklisting, a common practice in the mobile gaming industry of preventing the promotion of specific apps or apps belonging to specific categories. In most cases, blacklisting is done by advertisers due to issues of the low quality of customers in terms of CLV for specific apps or categories of apps (Kim 2020, Digital Limbo 2019). More frequently, blacklisting is used when apps want to avoid churn of high-value customers or when apps believe that for some advertisers, they can benefit from the advertising revenue yet keep their customers. Some market observers criticize the act of blacklisting. Specifically, it has been argued (a) that users will churn anyhow, and blacklisting just decreases their value because it limits the options for cross-promotion; (b) that a "blacklisting war" will decrease revenue to all sides; and (c) that apps can work instead on targeting that will ensure that just the right customer will get the ad (Digital Limbo 2019). While such claims can certainly be valid in some cases, our formal framework enables us to look at the situation differently, as summarized by the following proposition:

> **Proposition 4:** Blacklisting, a common tactic in the mobile game industry that prevents the cross-promotion of specific apps, will increase the likelihood of cross-promotion (either purchasing or swapping a user from a rival).

**Proof of Proposition 4:** Recall that though we have assumed a homogenous inside market, an alternative assumption is to assume a distribution in market quality. Formally this reflects in assuming inside market quality to follow a uniform distribution with $\alpha \sim U(a_{min}, a_{max})$ and thus define $\bar{\alpha}$ as the mean of this distribution $\bar{\alpha} = (a_{min} + a_{max})/2$. Blacklisting involves restrictions on swapping customers from the inside market whose quality is low, reflected in $\bar{\bar{\alpha}}$, and specifically, the app developer bans the low end of the distribution. Assuming blacklisting is effective, this means that there is a new lower bound of the distribution $\alpha_{min}^{bl} > a_{min}$. The implies a new mean of the distribution $\bar{\alpha}^{bl} > \bar{\bar{\alpha}}$. No other parameter is affected by this, including $\alpha_T$. From Table 6, we conclude that larger $\alpha$ implies a higher likelihood of satisfying the condition $\alpha \geq \alpha_T$, increasing the likelihood of cross-promotion. Note that while we consider the effect of blacklisting of specific apps, the final aim is to consider individual customers. In the absence of individual-level data, firms will blacklist apps based on the average value of their customers. Individual-level data will enable forms to differentiate between customers in this regard.

## 6. EXTENSIONS

**Outside Market Quality and *CAC***: Customer acquisition cost (*CAC*) is the cost paid to the likes of Facebook or Google that provide customer acquisition from the outside market whose quality is given by $\beta$. The quality of the outside market measures the ability of this market to target advertising to prospects that fit the game well. It can thus be thought of as conversion probability. *CAC* cannot be made endogenous as the game publishers are small and cannot influence, let alone control of the pricing mechanisms of Google and Facebook. However, the price of the good (advertising) should be related to its quality, and thus *CAC* should be related to $\beta$. We assume a log-linear relationship and thus for some parameter $\gamma$:

(17) $CAC = \text{constant} \cdot \beta^{\gamma}$

Somewhat counterintuitively, there are reasons to believe that the relationship between *CAC* and quality as measured, say by the conversion rate, is negative, that is $\gamma \leq 0$ (we will shortly assume that the constant of Equation 17 is one). First, from a theoretical point of view, note that if Facebook determines both quality and price, then there is no causality implied by the relationship of Equation 17, and the relationship might very well be negative[5]. Second, from an empirical point of view, one can look at the relationship between average Facebook advertising cost per conversion and the conversion rates in different industries (see Bond 2020 for a cross industry example). The correlation at the data in Bond (2020) between average Facebook ads cost per conversion, and the conversion rates is negative (-0.5). Moreover, if we take the same tables for Google Display Networks ads and Google Search ads (also in Bond 2020) and run a pooled regression of the three datasets using Equation 17, we find out that the regression fits the data relatively well (R-Squared of 70%) and that $\gamma = -0.52$, and significant at the 99% level.

There are at least two reasons for this negative correlation between *CAC* and conversion: First, Google and Facebook established a measure of ad quality, called Quality Score and Relevance Score, respectively, such that the higher the score, the higher the conversion rate and the lower is the ad costs (see Finn 2020)[6]. Second, CAC is computed retroactively by taking the

[5] To see this, assume a loglinear demand function of a firm that sets both quality ($q$) and price ($p$) simultaneously so that it maximizes profits given by: $\pi = (p - c)p^{-a}q^{b} - q^{2}/2$ where $c$ is the production costs, and $a$ and $b$ are price and quality elasticities. First-order conditions imply that $p = \text{constant} \cdot q^{\gamma}$, where $\gamma = (2 - b)/(1 - a)$. Second-order conditions imply that $\gamma < 0$.

[6] The ability to establish these nontransparent measures, and use them profitably, is undoubtedly one more demonstration of the substantial market power of these two firms.

entire advertising budget and dividing it by the total number of new users. Given a slight decline in conversion rate, the calculated *CAC* increases.

To replicate analyses of the equilibrium outcomes and Web Appendix B, without loss of generality, let the constant of Equation 17 be one and thus $CAC = \beta^{\gamma}$. The entire analysis goes thorough as is, except for Table 7 that now replaces Table 6. In addition, if $\gamma \leq 0$ the analysis remains the same without additional constraints, while if $\gamma \geq 0$ additional constraints are needed as follows: If $\gamma \leq 0$, then $\partial F/\partial\beta > 0$ and the analysis goes through. However, if $\gamma \geq 0$, we need a lower bound on *CLV* to ensure that $\partial F/\partial\beta > 0$, that is $CLV > \gamma\beta^{\gamma-1}$.

**Table 7: Market Equilibrium Outcomes when *CAC* depends on outside market quality $\boldsymbol{\beta}$***

| | | **Cross Promotion** | |
|---|---|---|---|
| | **Acquisition from the Outside Market** | **Purchase from Seller on the Inside Market** | **Swap with the Seller on the Inside Market** |
| Quality of the inside market ($\alpha$) | $\alpha \leq \alpha_T$ | $\alpha \geq \alpha_T$ | $\alpha \geq \alpha_T$ |
| Quality of the outside market ($\beta$) | $\beta \geq \beta_T$ | $F(\beta) \geq \alpha - \alpha_T$ | $F(\beta) \leq \alpha - \alpha_T$ |
| Costs to the buyer per one user | *CAC* | $PAC = (\alpha - \beta) \cdot CLV + \beta^{\gamma}$ | *RSV* |

* $F(\beta) = \beta - \beta^{\gamma}/CLV$. Note that since $\partial F/\partial\beta > 0$ it follows that the inequalities of $F(\beta)$ imply inequalities of $\beta$ in the same direction that is $F(\beta) \geq \alpha - \alpha_T$ implies $\beta \geq F^{-1}(\alpha - \alpha_T)$

The lower bound on *CLV* is a binding constraint for the rest of the propositions: In Web Appendix D, we replicate the sensitivity analyses of Web Appendix C, for the case where *CAC* and $\beta$ are related, where we show that all our four propositions can be replicated if $CLV > 1$ and $\gamma \leq 1$.

**Partial Resetting**: So far, we have assumed complete resetting of a swapped user, i.e., the swapped users fully reset their engagement in the new game to the same level that they began with in the old game. For example, in Game 3 in Table 2, the activity per user begins at about 27 minutes (average daily) in the first week and declines to about 16 minutes in the 8th week. Suppose the customer is swapped this week: We assumed that the resetting is such that now she starts playing the new game at complete resetting of 27 minutes per day. One could reasonably ask if we should expect some level of satiation across games, at least in the long tun, instead of a full reset? Thus, suppose that the resetting is partial, that is, the player in our example above, instead of resetting to the full 27 minutes, resets to a lower bound of $\alpha^{reset} \cdot 27$, where $0 \leq \alpha^{reset} \leq 1$. The receiving publisher now gets $\alpha^{reset}CLV$ instead of $CLV$.

However, note that we have already considered another bound in the CLV and that of similarity: To what extent does playing in one game serve as a useful screening mechanism for another game. The similarity between the games plays a role: If the seller's game is similar to the buyers' in terms of the characteristics and preference of the players, the seller can expect a higher quality customer $\alpha$, which will raise the effective $CLV$. Suppose we denote the fraction we have used so far ($\alpha$) because of screening $\alpha^{screen}$ and recall that we denoted the resetting fraction by $\alpha^{reset}$, then the publisher on the receiving end of swapping or selling would get a value of $\alpha^{new}CLV$, where $\alpha^{new} = \alpha^{screen} \cdot \alpha^{reset}$. The rest of the analysis now goes through with this new inside market quality $\alpha^{new}$ replacing $\alpha$. Since $\alpha^{new} \leq \alpha$ the net result would be less swapping and more selling, or if now the new inside quality is below the threshold, more purchasing on the outside market.

## 7. DISCUSSION

Historically marketers looked at customer acquisition from an outside market lens where buyers consider customer acquisition cost, conversion rate, and customer lifetime value when making acquisition decisions (Peters, Verhoef, and Krafft 2015). The emergence of an inside market where firms sell and swap customers, demands a broader view. The mobile gaming industry, where satiation coexists with efficient customer management abilities, is valuable for examining these new dynamics. In such environments, both buyers and sellers weigh the inside vs. outside market alternatives and within the inside market – selling vs. swapping. We demonstrate the broadness of this process and the dynamics that are to be considered. Using this approach, we provided several insights into the market condition under which cross-promotion is created, and if it does, the cases where selling or swapping is preferred. Next, we discuss some more general aspects that emerge from our analysis.

**The Optimal Engagement of Customers**: There is increasing recognition in the marketing literature of customer engagement and its contribution to profitability (Kumar and Pansari 2016; Gill, Sridhar, and Grewal 2017). While positive sentiment on the importance of engagement can certainly be justified, there is also a need to examine the optimal level of customer engagement given the costs associated with it. The case of satiation helps to shed some light on this issue. Satiation, which is a process of decreased engagement, can be affected by the firm in various design efforts, some of which can consider the behavioral findings in this area (Galak and Redden 2018).

Our analysis demonstrates that while there is an optimal level of satiation for the firm, one should consider how the interaction between sellers, buyers, and the outside market may affect it. While realizing the contribution of engaged customers, a realistic view of optimal engagement will help in better planning, and from a broader research view, why customers are not invested in or engaged with the product. In this regard, one of the more intriguing engagement enhancement techniques is dynamic difficulty adjustment (DDA) that adaptively changes a game to make it easier or more complex, depending on the players' state of mind, such as frustration or boredom (Pfau et al. 2020).

**A Broader View on Customer Profitability**: The shift to an inside market view provides an interesting angle on customer profitability and retention. The importance of avoiding churn (Ascarza et al., 2018) and the need to manage the tradeoff between investments in customer acquisition and retention (Reinartz et al., 2005) have generally taken a view of a single brand implicitly assuming the outside market is the only option the firm faces. Including inside markets requires a more nuanced approach as churning a customer might be profitable, as it generates the potential for acquiring a better replacement customer to a satiated one.

The emergence of inside markets also adds a different view on addressing unprofitable customers: It is generally accepted that firms may not want to retain some less profitable customers (Haenlein, Kaplan and Schoder 2006). Consequently, researchers have examined the market conditions and cost structures under which a firm may want to "fire" customers (Shin et al., 2012; Subramanian et al., 2014). We elucidate here that under satiation, even "bad customers" may have started as "good" and that the possibility of selling and swapping in an inside market is another alternative to deal with the less profitable customers.

**Customer Equity – Heterogeneity and Social Influence**: The inclusion of inside markets emphasizes the advantages of taking a customer equity look at customer profitability rather than that of an individual lifetime value perspective (Drèze and Bonfrer 2009). The firm maximizes the value of a group of customers that can be transferred and exchanged for other customers, not the value of the individuals *per-se*. To better understand the dynamics and enable a parsimonious analysis, we focused on transferring individuals. A customer equity perspective will allow examining better how heterogeneity among customers affects the dynamics of cross-promotion. When building the model, we assumed that market quality α and β are taken from a distribution. Understanding the shape of the distribution will enable a more informed decision on multiple cases of cross-promotion.

A notable source of heterogeneity stems from the temporal sorting mechanism of churn, where the users who stay longer may be different from users who churned early and have, for example, a higher expected retention probability or engagement (Fader and Hardie 2010). This sorting mechanism may also affect the level of satiation and expected quality of the users for the buyer. Our analysis does not deal with the measurement issues associated with assessing the value of the parameters. In practice, a Bayesian updating mechanism will be needed to update individual expectations based on the individual behavior with time. Our understanding of the effect of the churn sorting mechanism coupled with a change in the individual over time is still in its early stages (Fader et al. 2018), and a better understanding of the phenomenon will also help measurement in our context.

A second aspect of the move from an individual-level view to multiple users and customer equity is social value. Individuals create social value when they affect the lifetime value of others (Haenlein and Libai 2013). In gaming, the social value can be driven not only by word of mouth, but also by network effects that happen particularly in multi-player games and can be transparent to users through the popularity tables that encourage new users to adopt. Thus, when moving a user from game A to game B, the former may lose some social value, and the latter may gain some. Given the change in social value over the product life cycle (Haenlein and Libai 2013), it may be expected that the later the lifecycle of game A and earlier the life cycle of game B, both sides will find the deal more profitable to adopt. However, the precise measurement of the effect is not trivial. Adding social value considerations to cross-promotion analysis is a promising avenue for future research.

**Cross Promotion and Clusters**: It is interesting to see that our analysis of the utility of restricting cross-promotion (Proposition 4) is consistent with the emerging market structure in mobile game markets. As Figure 2 shows, swapping becomes increasingly likely the better the quality of the inside market. In situations where acquisition on the outside market is expensive – which is the case in the hyper-casual segment of the mobile game market – game developers will have an incentive to create an inside market of high quality to exclude other games that deem to be of lower quality.

One such strategy is to create games that are similar to those of competitors. A high degree of similarity implies that the screening performed by the seller is a good indicator of value for the buyer. Such similarity can be expected to lead to the emergence of clusters on the category level where multiple game developers are specialized in developing similar games. Empirical indication for this can be found when looking at *Candy Crush Saga*, one of the most successful mobile games. The popularity of *Candy Crush Saga* has resulted in the emergence of an entire industry based on the same game philosophy called tile-matching video games (Match 3 games), which includes dozens of games from different developers. This leads to the conclusion that in categories where the cost of acquiring customers on the outside market is high, game developers have an incentive to increase the similarity of games on the inside market, leading to the emergence of ***clusters*** of similar games among which users are cross-promoted (either purchased or swapped).

**Cross Promotion Outside the Mobile-Games Industry**: One could wonder why we do not see more inside market activities outside the gaming environment, given the ubiquity of satiation. We believe that technology plays an essential role in answering this question. For example, for Fast Moving Consumer Goods (FMCG), consumers are likely to be satiated after frequent consumption of the same brand, leading to variety-seeking behavior and brand switching (Kahn 1995; Wang and Shankar 2017). However, players in FMCG markets do not have the ability (yet) to track customers on an individual basis, track the exact usage of the product by the customer to make recommendations at the right time, or analyze profitability similarly to mobile games.

However, there are other markets that should be relevant. One example is the market for personalized content recommendations in online news outlets, such as Outbrains and Taboola, mentioned earlier is one. Other natural candidates come from fast-moving hedonic experiences like music streaming and YouTube videos where customers can be managed and possibly cross-promoted on a large scale. These present opportunities to develop inside markets. While satiation is undoubtedly a problem in these and other hedonic experience markets, it is still hard to find an equivalent to the gaming industry in the sophisticated real-time customer management that will identify satiation, assess lifetime value, and be able to cross-promote where needed efficiently. As more markets acquire these skills and abilities, we expect to observe the process of selling and swapping customers more relevant in an increasing number of markets.

## REFERENCES


Appel, Gil, Barak Libai, Eitan Muller, and Ron Shachar (2020), "On the Monetization of Mobile Apps," *International Journal of Research in Marketing*, 37, 93-107.

AppsFlyer (2020), "The State of Gaming App Marketing – 2020 edition," https://www.appsflyer.com/resources/reports/gaming-app-marketing/

Ascarza, Eva, Scott Neslin, Oded Netzer, Zachery Anderson, Peter Fader, Sunil Gupta, Bruce Hardie, Aurelie Lemmens, Barak Libai, David Neal, Foster Provost, and Rom Y. Schrift (2018), "In Pursuit of Enhanced Customer Retention Management: Review, Key Issues, and Future Directions," *Customer Needs and Solutions*, 5, 65-81.

Ascarza, Eva, Oded Netzer, and Julian Runge (2020), "The Twofold Effect of Customer Retention in Freemium Settings," SSRN

Banis, Davide (2018), "How This Startup Transforms Mobile Games into Veritable Gold Mines," TNW, February 5, https://thenextweb.com/insider/2018/02/06/first-user-acquisition-game-monetization-golden-rule-indie-game-developers/

Bolton, Ruth N., Katherine N. Lemon, and Peter C. Verhoef (2004), "The Theoretical Underpinnings of Customer Asset Management: A Framework and Propositions for Future Research," *Journal of the Academy of Marketing Science*, 32(3), 271-292.

Bond, Connor, (2020), "Conversion Rate Benchmarks: Find Out How YOUR Conversion Rate Compares, " https://www.wordstream.com/blog/ws/2019/08/19/conversion-rate-benchmarks

Cao, Jingcun, Pradeep K. Chintagunta, and Shibo Li (2021), "From Free to Paid: Monetizing a Non-advertising-based App," SSRN

Caro, Felipe and Victor Martínez-de-Albéniz (2012), "Product and Price Competition with Satiation Effects," *Management Science*, 58 (7), 1357-73.

Choi, Hana, Carl F. Mela, Santiago R. Balseiro, and Adam Leary (2020). "Online display Advertising Markets: A Literature Review and Future Directions," *Information Systems Research*, forthcoming.

Datta, Hannes, Bram Foubert, and Harald J. Van Heerde (2015), "The Challenge of Retaining Customers Acquired with Free Trials," *Journal of Marketing Research*, 52(2), 217-234.

Dellarocas, Chrysanthos, Zsolt Katona, and William Rand (2013), "Media, Aggregators, and the Link Economy: Strategic Hyperlink Formation in Content Networks," *Management Science*, 59 (10), 2360-79.

Digital Limbo (2019), "Digital Pill: Blacklist in Ad Monetization," January 19, https://www.digitalimbo.com/blacklist-in-ad-monetization/

Dogtiev, Artyom (2020a), "Top Mobile Ad Networks 2019," *Business of Apps*, https://www.businessofapps.com/guide/top-mobile-ad-networks/

Dogtiev, Artyom (2020b), "Cross Promotion Advertising Networks," *Business of Apps*, https://www.businessofapps.com/ads/cross-promotion/

Drèze, Xavier, and André Bonfrer (2009), "Moving from Customer Lifetime Value to Customer Equity," *Quantitative Marketing and Economics*, 7(3), 289-320.

Fader, Peter S., and Bruce GS Hardie (2010), "Customer-Base Valuation in a Contractual Setting: The Perils of Ignoring Heterogeneity, " *Marketing Science*, 29(1), 85–93.

Fader, Peter S., Bruce GS Hardie, Yuzhou Liu, Joseph Davin, and Thomas Steenburgh (2018), "'How to Project Customer Retention' Revisited: The Role of Duration Dependence, " *Journal of Interactive Marketing*, 43, 1–16

Fang, Eric, Robert W. Palmatier, and Rajdeep Grewal (2011), "Effects of Customer and Innovation Asset Configuration Strategies on Firm Performance," *Journal of Marketing Research*, 48(3), 587-602.

Finn, Allen (2020), "Facebook Relevance Score: 4 Key Facts to Know," https://www.wordstream.com/blog/ws/2017/09/14/facebook-relevance-score

Frid, Igal (2019), "How Hyper Casual Apps Are Changing the Way Games Monetize," *AppsFlyer*, November 19, https://www.appsflyer.com/blog/hyper-casual-gaming-ad-monetization/

Galak, Jeff, and Joseph P. Redden (2018), "The Properties and Antecedents of Hedonic Decline," *Annual Review of Psychology*, 69 (6.1-6.25).

Galak, Jeff, Justin Kruger, and George Loewenstein (2013), "Slow Down! Insensitivity to Rate of Consumption Leads to Avoidable Satiation," *Journal of Consumer Research*, 39 (5), 993-1009.

Gupta, Sunil, and Donald R. Lehmann (2003), "Customers as Assets," *Journal of Interactive Marketing*, 17(1), 9-24.

Haenlein, Michael, Andreas M. Kaplan, and Detlef Schoder (2006), "Valuing the Real Option of Abandoning Unprofitable Customers When Calculating Customer Lifetime Value," *Journal of Marketing*, 70 (3), 5-20.

Haenlein, Michael, and Barak Libai (2013), "Targeting Revenue Leaders for a New Product," *Journal of Marketing*, 77 (3), 65-80.

Han, Sang Pil, Sungho Park, and Wonseok Oh (2016), "Mobile App Analytics: A Multiple Discrete-Continuous Choice Framework," *MIS Quarterly*, 40 (4), 983-1008.

Hui, Sam K. (2017), "Understanding Repeat Playing Behavior in Casual Games Using a Bayesian Data Augmentation Approach," *Quantitative Marketing and Economics*, 15 (1), 29-55.

IronSource (2021) "Cross Promotion,"
https://developers.ironsrc.com/ironsource-mobile/air/cross-promotion/#step-1

Johnson, Michael D., and Fred Selnes (2004). "Customer Portfolio Management: Toward A Dynamic Theory of Exchange Relationships." *Journal of Marketing*, 68 (2), 1-17.

Kahn, Barbara E. (1995), "Consumer Variety-Seeking Among Goods and Services: An Integrative Review," *Journal of Retailing and Consumer Services*, 2 (3), 139-48.

Karnes, KC (2020), "Hyper-Casual Games: Mobile Gaming's Greatest Genre," CleverTap, https://clevertap.com/blog/hyper-casual-games/

Kim, Yohyun (2020), "Effects of Removing Blacklisting,", AdColony, https://www.adcolony.com/blog/2020/11/15/effects-of-removing-blacklisting/

Knott, Aaron, Andrew Hayes, and Scott A. Neslin (2002), "Next-Product-To-Buy Models for Cross-Selling Applications," *Journal of Interactive Marketing*, 16 (3), 59-75.

Lasaleta, Jannine D. and Joseph P. Redden (2018), "When Promoting Similarity Slows Satiation: The Relationship of Variety, Categorization, Similarity, and Satiation," *Journal of Marketing Research*, 55 (3), 446-57.

Lee, Gene Moo, Shu He, Joowon Lee, and Andrew B. Whinston (2020). "Matching Mobile Applications for Cross-Promotion," *Information Systems Research*, 31(3), 865-891.

Lerner, Ben (2019), "User Churn in Mobile Apps: Advertiser Blacklisting Benchmarks," Soomla.com, January 9, https://blog.soomla.com/2019/01/industry-first-user-churn-in-mobile-apps-advertiser-blacklisting-benchmarks-by-soomla.html

Lewis, Michael (2006), "Customer Acquisition Promotions and Customer Asset Value," *Journal of Marketing Research*, 43 (2), 195-203.

Li, Shibo, Baohong Sun, and Alan L. Montgomery (2011), "Cross-Selling the Right Product to the Right Customer at the Right Time," *Journal of Marketing Research*, 48 (4), 683-700.

Luz, Loren (2019), "What we learned from eMarketer's new in-game advertising report," IronSource, https://www.ironsrc.com/blog/emarketer-new-in-game-advertising-report/

Morgan, Blake (2019), "NOwnership, No Problem: An Updated Look at Why Millennials Value Experiences Over Owning Things,", Forbes, Jan 2.

Nelson, Leif D., Tom Meyvis, and Jeff Galak (2009), "Enhancing the Television-Viewing Experience through Commercial Interruptions," *Journal of Consumer Research*, 36 (2), 160-72.

Oba, Demi (2017), "*CLV* to *CAC* Ratio - Calculating and Understanding the Magic Metric," *Smile.io*, https://blog.smile.io/clv-cac-ratio-calculating-the-magic-metric/

Oestreicher-Singer, Gal, Barak Libai, Liron Sivan, Eyal Carmi, and Ohad Yassin (2013), "The Network Value of Products," *Journal of Marketing*, 77 (3), 1-14.

Peters, Kay, Peter C. Verhoef, and Manfred Krafft (2015), "Customer Acquisition Strategies: A Customer Equity Management Perspective," In V. Kumar and Denish Shah (eds.) *Handbook of Research on Customer Equity in Marketing*. Edward Elgar Publishing.

Pfau, Johannes, Jan David Smeddinck, and Rainer Malaka, (2020), "Enemy Within: Long-Term Motivation Effects of Deep Player Behavior Models for Dynamic Difficulty Adjustment," *Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems*, 1-10.

Popescu, Clara (2020), "A Guide to Cross-Promotion: How to Increase Installs for Your Mobile Game for Free," *App/Agent*, April 22, https://www.appagent.co/blog/2020/04/22/a-guide-to-cross-promotion-how-to-increase-installs-for-your-mobile-game-for-free/

Prins, Remco, and Peter C. Verhoef (2007), "Marketing Communication Drivers of Adoption Timing of a New E-Service Among Existing Customers," *Journal of Marketing*, 71 (2), 169-83.

Rankmyapp (2019), "To Use Or Not To Use Cross Promoting Strategies for Mobile App Promotion," April 19, https://www.rankmyapp.com/market/to-use-or-not-to-use-cross-promoting-strategies-for-mobile-app-promotion/

Reinartz, Werner, Jacquelyn Thomas, and V. Kumar (2005), "Balancing Acquisition and Retention Resources to Maximize Customer Profitability," *Journal of Marketing*, 69 (1), 63-79.

Rust, Roland T., Katherine N. Lemon, and Valarie A. Zeithaml (2004), "Return on Marketing: Using Customer Equity to Focus Marketing Strategy," *Journal of Marketing*, 68 (1), 109-127.

Sajeesh, S. and Jagmohan S. Raju (2010), "Positioning and Pricing in a Variety Seeking Market," *Management Science*, 56 (6), 949-61.

Salz, Peggy Anne (2015), "Tapdaq Empowers App Publishers To Create Community, Take Charge of User Acquisition," Forbes, March 24.

Seufert, Eric (2019), "How Large is the Mobile Gaming Advertising Market," *Medium*, https://medium.com/@eric_seufert/how-large-is-the-mobile-gaming-advertising-market-9729e1551a8a

Sevilla, Julio, Jiao Zhang, and Barbara E. Kahn (2016), "Anticipation of Future Variety Reduces Satiation from Current Experiences," *Journal of Marketing Research*, 53 (6), 954-68.

Schmitz, Christian, You-Cheong Lee, and Gary L. Lilien (2014), "Cross-Selling Performance in Complex Selling Contexts: An Examination of Supervisory- and Compensation-Based Controls," *Journal of Marketing*, 78 (3), 1-19.

SensorTower (2021), "Global App Spending Approached $65 Billion in the First Half of 2021, Up More Than 24% Year-Over-Year," *SensorTower*, June 28, https://sensortower.com/blog/app-revenue-and-downloads-1h-2021

Shi, Zijun, Kaifu Zhang, and Kannan Srinivasan (2019), "Freemium as an Optimal Strategy for Market Dominant Firms," *Marketing Science*, 38(1), 150-169.

Shin, Jiwoong, K. Sudhir, and Dae-Hee Yoon (2012), "When to "Fire" Customers: Customer Cost-Based Pricing," *Management Science*, 58 (5), 932-47.

Silver, Davis (2020), "Top 51 Mobile App and Gaming Statistics 2021," *IronSource*, https://www.is.com/blog/resources/top-51-mobile-app-and-gaming-statistics/

Song, Yicheng, Nachiketa Sahoo, and Elie Ofek (2019), "When and How to Diversify - A Multicategory Utility Model for Personalized Content Recommendation." *Management Science*, 65 (8), 3737-3757.

Stephen, Andrew T. and Olivier Toubia (2010), "Deriving Value from Social Commerce Networks," *Journal of Marketing Research*, 47 (2), 215-28.

Subramanian, Upender, Jagmohan S. Raju, and Z. John Zhang (2014), "The Strategic Value of High-Cost Customers," *Management Science*, 60 (2), 494-507.

Villanueva, Julian, Shijin Yoo, and Dominique M. Hanssens (2008), "The Impact of Marketing-Induced Versus Word-Of-Mouth Customer Acquisition on Customer Equity Growth," *Journal of Marketing Research*, 45(1), 48-59.

Wong, Steven (2016), "How Disney Mobile Is Crossing into the Future of Entertainment," http://www.alistdaily.com/strategy/disney-mobile-gaming-crosses-future-entertainment/

**Web Appendix A**: Optimal Satiation

The firm is faced with optimizing the profit function:

(1) $$\pi = \frac{rg}{1+d-\delta\mu r} - \frac{1}{2}c(\delta\mu)^2$$

Taking the first derivative of this function shows that:

(2) $$\frac{\partial\pi}{\partial(\delta\mu)} = \frac{gr^2}{(1+d-\delta\mu r)^2} - c\delta\mu$$

(3) $$\frac{\partial\pi}{\partial\delta} = \mu\left(\frac{gr^2}{(1+d-\delta\mu r)^2} - c\delta\mu\right) = \mu\frac{\partial\pi}{\partial(\delta\mu)}$$

(4) $$\frac{\partial\pi}{\partial\mu} = \delta\left(\frac{gr^2}{(1+d-\delta\mu r)^2} - c\delta\mu\right) = \delta\frac{\partial\pi}{\partial(\delta\mu)}$$

It is easy to see that the first derivative of the profit function with respect to the two satiation parameters and their product is identical, but for a scaling factor. This implies that optimizing with respect to either parameter leads to the same result and thus the optimum is $\delta = \mu$. In the following, we optimize with respect to the product of both parameters $\mu\delta$. For convenience, we substitute:

(5) $$x = \mu\delta$$

Leading to:

(6) $$\frac{\partial\pi}{\partial x} = \frac{gr^2}{(1+d-xr)^2} - cx = 0$$

Equation 6 can be simplified to:

(7) $$\frac{gr^2}{c} = x(1+d)^2 - 2x^2r(1+d) + x^3r^2$$

Discarding the trivial solution $r = 0$, dividing by $r^2$ and rearranging terms results in:

(8) $$x^3 - 2\frac{1+d}{r}x^2 + \left(\frac{1+d}{r}\right)^2 x - \frac{g}{c} = 0$$

We substitute:

(9) $$\beta = \frac{1+d}{r}$$

Leading to:

(10) $$x^3 - 2\beta x^2 + \beta^2 x - \frac{g}{c} = 0$$

Equation 10 provides us with the first-order condition (FOC) function:

(11) $$FOC(x) = x^3 - 2\beta x^2 + \beta^2 x - \frac{g}{c} = x(x-\beta)^2 - \frac{g}{c}$$

Taking the first derivative of FOC gives:

(12) $\frac{\partial FOC}{\partial x} = 3x^2 - 4\,\beta x + \beta^2 = (\beta - x)(\beta - 3x)$

**Figure A1:** Exemplary FOC

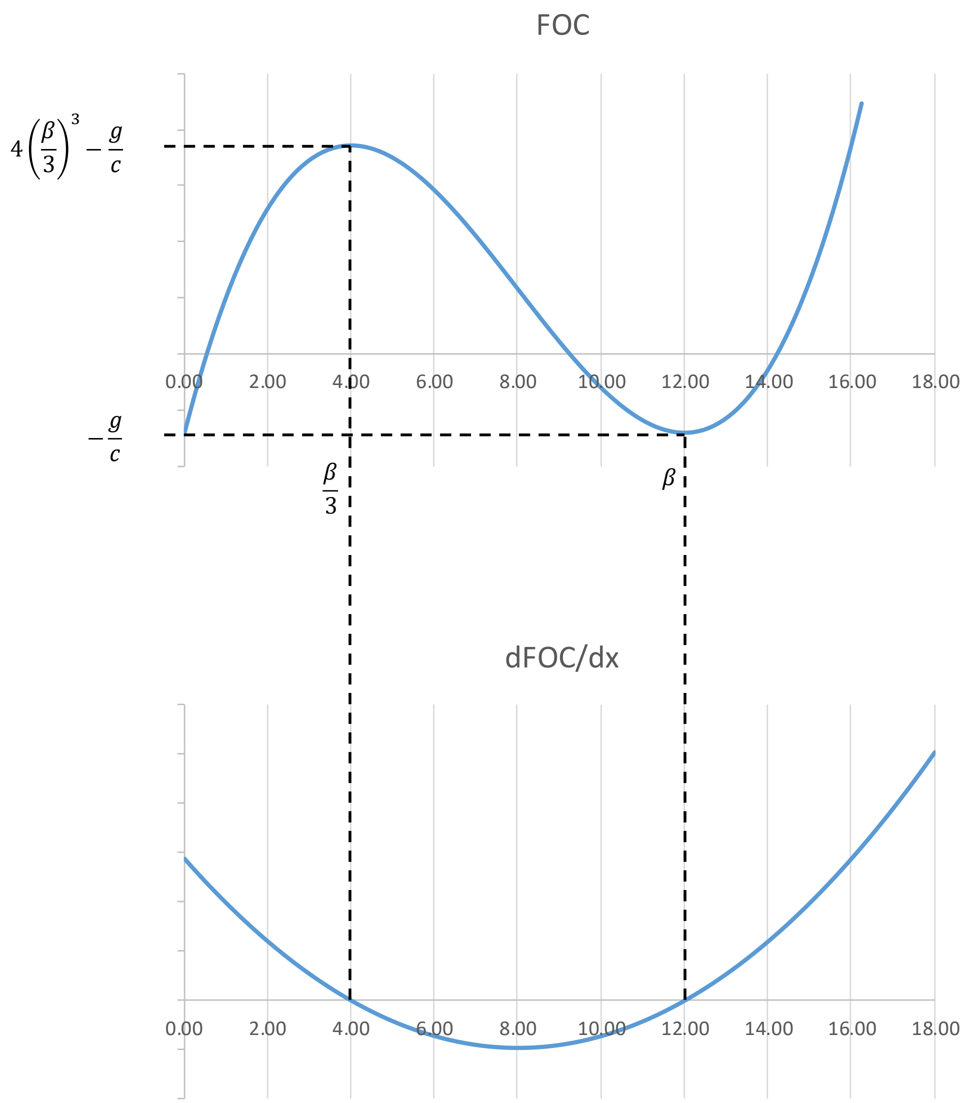



Figure A1 shows an exemplary FOC and its derivative for $r = 0.10, g = 0.70, d = 0.20,$ and $c = 0.01$. We see that FOC has the following four properties:

1. FOC starts at the *negative* value $FOC(0) = -\frac{g}{c}$.
2. In the interval $0 < x < \frac{\beta}{3}$ FOC *increases* since the first-order derivative of FOC is positive. At $x = \frac{\beta}{3}$ FOC reaches a *maximum* since the first-order derivative of FOC is zero. The maximum is reached at the value $FOC\left(\frac{\beta}{3}\right) = 4\left(\frac{\beta}{3}\right)^3 - \frac{g}{c}$.

3. In the interval $\frac{\beta}{3} < x < \beta$ FOC *decreases* since the first-order derivative of FOC is negative. At $x = \beta$, FOC reaches a *minimum* since the first-order derivative of FOC is zero. The minimum is reached at the negative value $FOC(\beta) = FOC(0) = -\frac{g}{c}$.
4. In the interval $x > \beta$ FOC *increases* since the first-order derivative of FOC is positive.

These properties imply that FOC has a first possible zero point $x_1$ in the interval $0 < x_1 < \frac{\beta}{3}$ and a second possible zero point $x_2$ in the interval $\frac{\beta}{3} < x_2 < \beta$, but only if $4\left(\frac{\beta}{3}\right)^3 - \frac{g}{c} \geq 0$ which is equal to $c \geq \frac{g}{4}\left(\frac{3}{\beta}\right)^3$. FOC has a third possible zero point $x_3$ in the interval $\beta < x_3$.
Regarding $x_3$, note that $\beta = \frac{1+d}{r} > 1$, since $r, d \leq 1$, and $\beta = 1$ requires $d = 0 \; and \; r = 1$.
Given that $x$ is the product of the two satiation parameters $\delta \; and \; \mu$, which are both less than 1, $x_3$ cannot be an optimum of the profit function.
To identify which of the remaining two zero points $x_1$ and $x_2$ optimizes the profit function, we look at the second derivative of the profit function:

(13) $\frac{\partial^2 \pi}{\partial^2 x} = \frac{2gr^3}{(1+d-xr)^3} - c$

Equation 13 can be reformulated as follows:

(14) $\frac{\partial^2 \pi}{\partial^2 x} = \frac{gr^2}{(1+d-xr)^2} \frac{2r}{(1+d-xr)} - c$

We know that at the optimum point $(x_i)$ the first derivative of the profit function is zero, which implies $\frac{gr^2}{(1+d-x_i r)^2} = cx_i$. The second-order derivative at the optimum point $x_i$ therefore becomes:

(15) $\frac{\partial^2 \pi}{\partial^2 x}(x_i) = cx_i \frac{2r}{(1+d-x_i r)} - c$

Equation 15 can be reformulated as follows:

(16) $\frac{\partial^2 \pi}{\partial^2 x}(x_i) = c \frac{3rx_i - (1+d)}{1+d-x_i r}$

For $x_i$ to be a maximum of the profit function, the first-order derivative must be zero (equal to FOC=0), and the second-order derivative (Equation 16) must be negative. It is easy to see that this either implies $x_i > \beta$ (in which case the numerator is positive and the denominator is negative) or $x_i < \frac{\beta}{3}$ (in which case the numerator is negative and the denominator positive). As established above $x_i > \beta$ cannot be a feasible solution since $\beta > 1$. Hence the profit function can only have a maximum if $x_i < \frac{\beta}{3}$, thus the only zero point which maximizes profit is $x_1$. To obtain a close form expression for $x_1$ we need to find the smallest solution to the cubic equation shown in Equation 10. In Equation 10, we substitute:

$$(17)\quad x = t + \frac{2}{3}\beta$$

This results in:

$$(18)\quad \left(t + \frac{2}{3}\beta\right)^3 - 2\beta\left(t + \frac{2}{3}\beta\right)^2 + \beta^2\left(t + \frac{2}{3}\beta\right) - \frac{g}{c} = 0$$

Equation 18 can be simplified to the depressed cubic:

$$(19)\quad t^3 - \frac{\beta^2}{3}t + \frac{2}{27}\beta^3 - \frac{g}{c} = 0$$

$$(20)\quad t^3 - 3\left(\frac{\beta}{3}\right)^2 t + 2\left(\frac{1}{27}\beta^3 - \frac{g}{2c}\right) = 0$$

According to Zucker (2008), Equation 3, the solution of Equation 20 is:

$$(21)\quad t_k = 2\frac{\beta}{3}\cos\left[\frac{1}{3}\arccos\left[\frac{g}{2c}\left(\frac{3}{\beta}\right)^3 - 1\right] + \frac{2\pi k}{3}\right] \text{ for k=0,1,2}$$

Combing Equation 17 and Equation 21 gives:

$$(22)\quad x_k = \frac{2}{3}\beta\left[\cos\left(\frac{1}{3}\arccos\left[\frac{g}{2c}\left(\frac{3}{\beta}\right)^3 - 1\right] + \frac{2\pi k}{3}\right) + 1\right] \text{ for } k = 0,1,2$$

Given the above, we are only interested in the smallest of the three possible solutions of Equation 22. We later show that this is the solution for which $k = 1$, and hence:

$$(23)\quad x_1 = \frac{2}{3}\beta\left[\cos\left(\frac{1}{3}\arccos\left[\frac{g}{2c}\left(\frac{3}{\beta}\right)^3 - 1\right] + \frac{2\pi}{3}\right) + 1\right]$$

We now separate two cases: If $\frac{\beta}{3} \leq 1$ equal to $r \geq \frac{1+d}{3}$, then $x_1$ is a feasible solution since $x_1 < \frac{\beta}{3} \leq 1$. If $\frac{\beta}{3} > 1$ equal to $r < \frac{1+d}{3}$, then we note that $FOC(0) = -\frac{g}{c} < 0$ and $FOC(1) = (1-\beta)^2 - \frac{g}{c}$. It is easy to see that $FOC(1) > 0$ if $c > \frac{g}{(1-\beta)^2}$. Hence as long as c is sufficiently large, we can always ensure that FOC has a zero point in the interval [0;1]. This results in Proposition A1 (recall that $x = \mu\delta$):

> **Proposition A1**: When the firm chooses the satiation in gross profit margins ($\delta$) and retention probability ($\mu$) subject to a convex cost structure, then when $c > c_1$ there exists

a unique solution $(\delta^*)^2 = (\mu^*)^2 = \frac{2}{3}\beta\left[\cos\left(\frac{1}{3}\arccos\left[\frac{g}{2c}\left(\frac{3}{\beta}\right)^3 - 1\right] + \frac{2\pi}{3}\right) + 1\right]$ that maximizes profit. When $c \leq c_1$ the firm chooses $\delta^* = \mu^* = 1$. If $r \geq \frac{1+d}{3}$, then $c_1 = \frac{g}{4}\left(\frac{3}{\beta}\right)^3$, while if $r \leq \frac{1+d}{3}$, then $c_1 = \frac{g}{(1-\beta)^2}$.[7]

There are two additional parts of the proof: First, that the smallest solution of Equation 22 is the one with $k = 1$, and further that when $c \leq c_1$, the firm indeed sets optimal satiation to 1, and its profits are non-negative.

We start by showing that the smallest solution of Equation 22 is the one with $k = 1$. According to Zucker (2008)[8], Equation 3, the solution of the cubic equation: (24) is Equation (25)

(24) $t^3 + 3pt + 2q = 0$

(25) $r_{1,2,3} = 2\sqrt{-p}\cos\left(\frac{\theta+2k\pi}{3}\right) = 2\sqrt{-p}\cos\left(\frac{\theta}{3} + \frac{2k\pi}{3}\right)$ for k=0,1,2

where

(26) $\cos(\theta) = \frac{-q}{\sqrt{-p^3}}$

Knowing that:

(27) $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

we can express Equation 25 as:

(28) $r_{1,2,3} = 2\sqrt{-p}\left[\cos\left(\frac{\theta}{3}\right)\cos\left(\frac{2k\pi}{3}\right) - \sin\left(\frac{\theta}{3}\right)\sin\left(\frac{2k\pi}{3}\right)\right]$

Using $k = 0{,}1{,}2$ yields the three solutions

(29) $r_{k=0} = 2\sqrt{-p}\left[\cos\left(\frac{\theta}{3}\right)\cos(0) - \sin\left(\frac{\theta}{3}\right)\sin(0)\right] = 2\sqrt{-p}\cos\left(\frac{\theta}{3}\right)$

(30) $r_{k=1} = 2\sqrt{-p}\left[\cos\left(\frac{\theta}{3}\right)\cos\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\theta}{3}\right)\sin\left(\frac{2\pi}{3}\right)\right] = 2\sqrt{-p}\left[-\frac{1}{2}\cos\left(\frac{\theta}{3}\right) - \frac{\sqrt{3}}{2}\sin\left(\frac{\theta}{3}\right)\right]$

(31) $r_{k=2} = 2\sqrt{-p}\left[\cos\left(\frac{\theta}{3}\right)\cos\left(\frac{4\pi}{3}\right) - \sin\left(\frac{\theta}{3}\right)\sin\left(\frac{4\pi}{3}\right)\right] = 2\sqrt{-p}\left[-\frac{1}{2}\cos\left(\frac{\theta}{3}\right) + \frac{\sqrt{3}}{2}\sin\left(\frac{\theta}{3}\right)\right]$

It is easy to see that $r_{k=1} < r_{k=0}$ and $r_{k=1} < r_{k=2}$ is equal to:

---

[7] As the inequality of $r$ and $(1 + d)/3$ is a weak inequality in both conditions, one can ask about the level of $c_1$ when they are equal: When $r = (1 + d)/3$ (that is $\beta = 3$), then $c_1 = g/4$ in both cases.

[8] Zucker, I.J. (2008), "The cubic equation – A new look at the irreducible case," *The Mathematical Gazette*, 92 (524), pp. 264-68.

(32) $3\cos\left(\frac{\theta}{3}\right) > -\sqrt{3}\sin\left(\frac{\theta}{3}\right)$ and $\sin\left(\frac{\theta}{3}\right) > 0$

Given Equation 26, we know that $0 \leq \theta \leq \pi$ or $0 \leq \frac{\theta}{3} \leq \frac{\pi}{3}$.

This implies $\frac{1}{2} \leq \cos\left(\frac{\theta}{3}\right) \leq 1$ and $0 \leq \sin\left(\frac{\theta}{3}\right) \leq \frac{\sqrt{3}}{2}$. Equation 32 is therefore fulfilled with proves that the smallest solution for the cubic equation is given for $k = 1$.

We also have to show that when $c \leq c_1$, the firm indeed sets the optimal satiation to 1, and its profits are non-negative. Plugging the value of 1 into the profit function yields:

(33) $\pi = \frac{rg}{1+d-r} - \frac{1}{2}c$

Requiring non-negative profits implies that $c \leq c_2$ where the latter is given by:

(34) $c_2 = \frac{2g}{\beta-1}$

According to Proposition A1, we have two cases:

<u>Case 1</u>: $r \leq \frac{1+d}{3}$, equal to $\beta \geq 3$

If $\beta \geq 3$ then $x_1 < 1 \leq \frac{\beta}{3}$ and if $c \leq c_1$ then $FOC(1) < 0$. Hence the derivative of the profit function is negative in the entire interval $0 < x_1 < 1$. This implies that profit increase in x, and the firm should set x to the highest value, corresponding to optimal satiation of 1. We now have to show that $c_2 = \frac{2g}{\beta-1} > \frac{g}{(1-\beta)^2} = c_1$, which translates to $2(\beta - 1) > 1$ or $\beta > \frac{3}{2}$. This is given since $\beta \geq 3$.

<u>Case 2</u>: $r \geq \frac{1+d}{3}$, equal to $\beta \leq 3$

If $\beta \leq 3$ then $x_1 < \frac{\beta}{3} \leq 1$ and if $c \leq c_1$ then $FOC\left(\frac{\beta}{3}\right) < 0$. Hence the derivative of the profit function is negative in the entire interval $0 < x_1 < \frac{\beta}{3}$. This implies that profit increase in x, and the firm should set x to the highest value, corresponding to optimal satiation of 1. We now have to show that $c_2 = \frac{2g}{\beta-1} > \frac{g}{4}\left(\frac{3}{\beta}\right)^3 = c_1$, which translates to $8\beta^3 - 27\beta + 27 > 0$. A simple plot of $L(\beta) = 8\beta^3 - 27\beta + 27$ shows that $L(\beta) > 0$ for $\beta \geq 0$.

Lastly, we can find the sensitivity of the optimal solution to the parameters as follows: From Proposition 1 and Equations (11) and (12), we know:

(35) $$\alpha_T = (\delta^* \mu^*)^T$$

(36) $$\beta_T = \frac{CAC}{rg}(1 + d - \delta^* \mu^* r)$$

(37) $$(\delta^*)^2 = (\mu^*)^2 = 2\,\frac{1+d}{3r}\left[\cos\left(\frac{1}{3}\arccos\left[\frac{g}{2c}\left(\frac{3r}{1+d}\right)^3 - 1\right] + \frac{2\pi}{3}\right) + 1\right]$$

We first check the sensitivity of the optimal satiation ($\delta^* = \mu^*$) concerning the parameters *g* and *c*. From Equations 11 and 12 we know that:

(38) $$FOC(x) = x\left(x - \frac{1+d}{r}\right)^2 - \frac{g}{c}$$

(39) $$\frac{\partial FOC}{\partial x} = \left(\frac{1+d}{r} - x\right)\left(\frac{1+d}{r} - 3x\right)$$

Equations 38 and 39 show that a change in *g* and *c* does not change the shape of FOC. The derivative of FOC (Equation 39) does not depend on *g* and *c* (but only on *d* and *r*). A change in *g* and *c* only shifts FOC vertically by changing the ratio $\frac{g}{c}$. Specifically, as shown in Figure A1, FOC shifts up if *g* decreases and if *c* increases. It is easy to see that an upwards shift of FOC means FOC cuts the x-axis at a lower point. Hence $\frac{\partial \delta}{\partial g} \geq 0$ and $\frac{\partial \delta}{\partial c} \leq 0$

Regarding the sensitivity of the optimal satiation ($\delta^* = \mu^*$) concerning the retention rate *r*, we use numerical derivatives. For this, we draw one million values for each parameter *g*, *d*, *c*, and *r* out of a uniform distribution between 0 and 1. We then determine the optimal satiation for this parameter set and the same parameters, replacing *r* by $r + 0.01$. We see that the derivative consistently increases with *r*. Hence $\frac{\partial \delta}{\partial r} \geq 0$

## **Web Appendix B**: Equilibrium Conditions

### 1. When does selling occur, and at which price?

We observe a purchase/ sale if the buyer prefers purchasing over acquiring and swapping, and the seller prefers selling over keeping and swapping. This leads to the following four conditions:

Selling-Condition 1: The buyer prefers purchasing over acquiring:

(1) $\alpha CLV - PAC + RSV \geq \beta \cdot CLV - CAC + RSV$

(2) $\beta \leq \alpha - \frac{PAC-CAC}{CLV}$

Selling-Condition 2: The buyer prefers purchasing over swapping:

(3) $\alpha CLV \leq \alpha CLV - PAC + RSV$

(4) $PAC \leq RSV$

Selling-Condition 3: The seller prefers selling over keeping:

(5) $PAC + \beta \cdot CLV - CAC \geq RSV$

(6) $\beta \geq \frac{RSV+CAC-PAC}{CLV}$

Selling-Condition 4: The seller prefers selling over swapping:

(7) $\alpha CLV \leq PAC + \beta \cdot CLV - CAC$

(8) $\beta \geq \alpha - \frac{PAC-CAC}{CLV}$

Selling-Condition 1 and Selling-Condition 4 imply that selling only takes place if:

(9) $\beta = \alpha - \frac{PAC-CAC}{CLV}$

(10) $PAC = (\alpha - \beta) \cdot CLV + CAC = \alpha \cdot CLV - (\beta \cdot CLV - CAC)$

Plugging the equilibrium *PAC* of Equation 10 into Selling-Condition 2 gives:

(11) $(1 - \beta) \cdot CLV + CAC - (1 - \alpha)CLV \leq RSV$

(12) $\beta \geq \alpha - \frac{RSV-CAC}{CLV}$

Plugging the equilibrium *PAC* into Selling-Condition 3 gives:

(13) $\beta \geq \frac{RSV+CAC-\alpha CLV+\beta CLV-CAC}{CLV}$

(14) $RSV \leq \alpha CLV$

Thus, we observe a purchase/ sale in equilibrium if $\beta \geq \alpha - \frac{RSV-CAC}{CLV}$ under the condition that $RSV \leq \alpha CLV$. The purchase acquisition cost at equilibrium is $PAC = (\alpha - \beta) \cdot CLV + CAC$.

### 2. When does swapping occur?

Similar to above, swapping occurs under the following four conditions:

Swap-Condition 1: The buyer prefers swapping over acquiring:

(15) $\alpha CLV \geq \beta \cdot CLV - CAC + RSV$

(16) $\beta \leq \alpha - \frac{RSV-CAC}{CLV}$

Swap-Condition 2: The buyer prefers swapping over purchasing:

(17) $\alpha CLV \geq \alpha CLV - PAC + RSV$

(18) $PAC \geq RSV$

Swap-Condition 3: The seller prefers swapping over keeping:

(19) $\alpha CLV \geq RSV$

Swap-Condition 4: The seller prefers swapping over selling:

(20) $\alpha CLV \geq PAC + \beta \cdot CLV - CAC$

(21) $\beta \leq \alpha - \frac{PAC-CAC}{CLV}$

Note that if Swap-Condition 4 holds and Swap-Condition 2 holds, then Swap-Condition 1 holds as well, since:

(22) $\alpha - \frac{PAC-CAC}{CLV} \leq \alpha - \frac{RSV-CAC}{CLV}$, if

(23) $PAC \geq RSV$

We now plug in the expression of equilibrium *PAC* from Equation 10 into Swap-Condition 2:

(24) $(\alpha - \beta) \cdot CLV + CAC \ \geq RSV$

(25) $\beta \leq \alpha - \frac{RSV-CAC}{CLV}$

Plugging in the expression of equilibrium *PAC* from Equation 10 into Swap-Condition 4:

(26) $\alpha CLV \geq (\alpha - \beta) \cdot CLV + CAC + \beta \cdot CLV - CAC$

(27) $0 \geq 0$ that is satisfied.

All of this implies that if $\beta \leq \alpha - \frac{RSV-CAC}{CLV}$ and $RSV \leq \alpha CLV$ then the equilibrium is swap.

### 3. When does acquisition occur?

Acquiring-Condition 1: The buyer prefers acquiring over purchasing:

(28) $\alpha CLV - PAC + RSV \leq \beta \cdot CLV - CAC + RSV$

(29) $\beta \geq \alpha - \frac{PAC-CAC}{CLV}$

Acquiring-Condition 2: The buyer prefers acquiring over swapping:

(30) $\alpha CLV \leq \beta \cdot CLV - CAC + RSV$

(31) $\beta \geq \alpha - \frac{RSV-CAC}{CLV}$

Acquiring-Condition 3: The seller prefers keeping over selling:

(32) $PAC + \beta \cdot CLV - CAC \leq RSV$

(33) $\beta \leq \frac{RSV - PAC + CAC}{CLV}$

Acquiring-Condition 4: The seller prefers keeping over swapping:

(34) $\alpha CLV \leq RSV$

Acquiring-Condition 5: The net benefit of acquiring for the buyer is positive:

(35) $\beta \cdot CLV - CAC \geq 0$

(36) $\beta \geq \frac{CAC}{CLV}$

We now plug in the equilibrium *PAC* from Equation 10 into Acquiring-Condition 1:

(37) $\alpha CLV - (\alpha - \beta) \cdot CLV - CAC \leq \beta \cdot CLV - CAC$ or $0 \leq 0$, which is satisfied.

We now plug in the equilibrium *PAC* from Equation 10 into Acquiring-Condition 3:

(38) $(\alpha - \beta) \cdot CLV + CAC + \beta \cdot CLV - CAC \leq RSV$

(39) $RSV \geq \alpha CLV$

Note that if Acquiring-Condition 4 holds and Acquiring-Condition 5 holds, then Acquiring-Condition 2 is fulfilled, since:

(40) $\frac{CAC}{CLV} \geq \alpha - \frac{RSV - CAC}{CLV}$, if $RSV \geq \alpha CLV$

We have two conditions on $\beta$:

(41) $\beta \geq \alpha - \frac{RSV - CAC}{CLV}$ and $\beta \geq \frac{CAC}{CLV}$

It is easy to see that:

(42) $\alpha - \frac{RSV - CAC}{CLV} \leq \frac{CAC}{CLV}$ if $CLV\alpha \leq RSV$

Therefore:

(43) $\alpha - \frac{RSV - CAC}{CLV} \leq \frac{CAC}{CLV} \leq \beta$

This shows that acquisition takes place if $\beta \geq \frac{CAC}{CLV}$ under the condition that $\alpha CLV \leq RSV$

Finally, we observe no action of any kind (i.e., neither purchase/ sale, nor swap or acquisition) in the equilibrium, when the buyer prefers acquiring over purchasing and swapping, and the seller prefers keeping over selling and swapping, but the net benefit of acquisition for the seller is *negative*. It is easy to see that this is equivalent to the previous case with a change in the last condition. Hence, no action occurs in equilibrium if:

(44) $\beta \leq \frac{CAC}{CLV}$ and $\alpha \leq \frac{RSV}{CLV}$

## **Web Appendix C:** Sensitivity Analysis

Figure 2 in the text shows that all equilibrium outcomes correspond to areas in a square defined by $\alpha$ and $\beta$. Since both $\alpha$ and $\beta$ are between 0 and 1, the total size of that square is 1. We can, therefore, interpret the size of each area as a measure of how likely a specific outcome is to occur. Given this logic, we define A as the likelihood of observing acquisition in equilibrium, P as the likelihood of observing purchase/ sale at *PAC* at equilibrium, and S as the likelihood of observing swap at equilibrium. Besides, we define $CP = P + S$ as the likelihood of observing any form of cross-promotion (either purchase/ sale or swap) in equilibrium.

**<u>Case 1</u>: $\boldsymbol{\beta_T \leq \alpha_T}$ or $\boldsymbol{CAC \leq RSV}$**

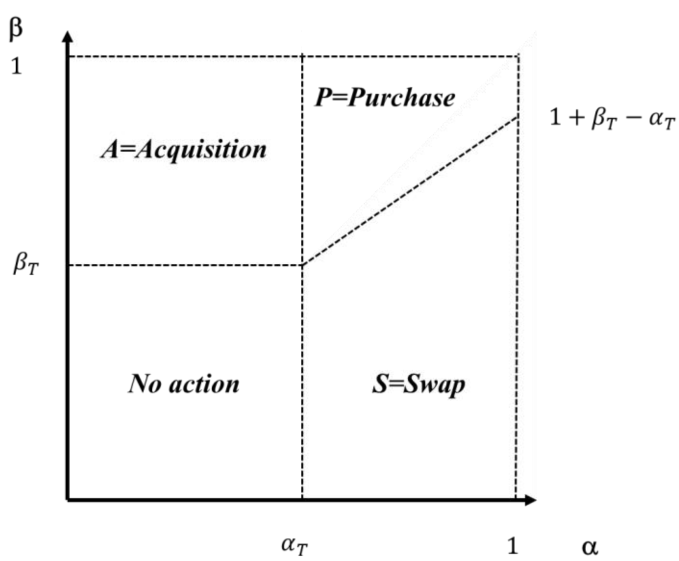
If $\beta_T \leq \alpha_T$ then $1+\beta_T - \alpha_T \leq 1$, and it is easy to see that:

(1) $A = \alpha_T(1 - \beta_T)$

(2) $S = \beta_T(1 - \alpha_T) + 0.5(1 - \alpha_T)(1 + \beta_T - \alpha_T - \beta_T) = \beta_T(1 - \alpha_T) + 0.5(1 - \alpha_T)^2$

(3) $P = (1 - \alpha_T) - S = (1 - \alpha_T)(1 - \beta_T) - 0.5(1 - \alpha_T)^2$

(4) $CP = P + S = 1 - \alpha_T$




**<u>Case 2</u>: $\boldsymbol{\beta_T > \alpha_T}$ or $\boldsymbol{CAC > RSV}$**

If $\beta_T > \alpha_T$ then $1+\beta_T - \alpha_T > 1$

The intersection of the line $\alpha + \beta_T - \alpha_T$ with $\beta = 1$ is at the point:

(5) $\alpha + \beta_T - \alpha_T = 1$ or $\alpha = 1 - \beta_T + \alpha_T$

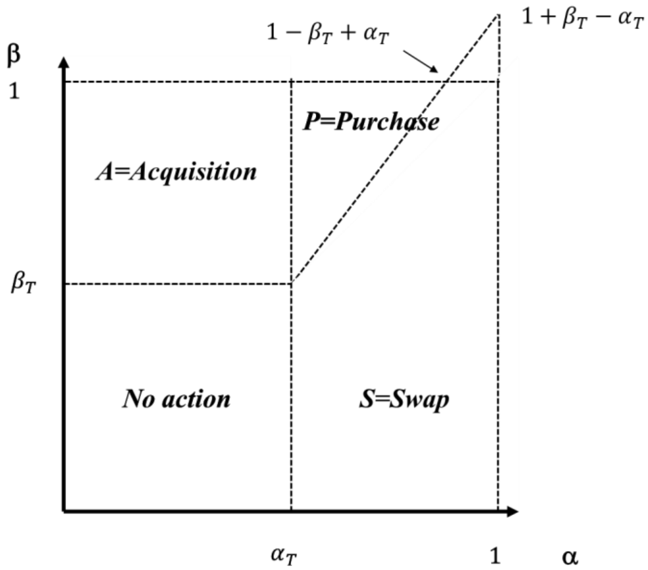
It is easy to see that:

(6) $A = \alpha_T(1 - \beta_T)$

(7) $P = 0.5(1 - \beta_T + \alpha_T - \alpha_T)(1 - \beta_T) = 0.5(1 - \beta_T)^2$

(8) $S = (1 - \alpha_T) - 0.5(1 - \beta_T)^2$

(9) $CP = 1 - \alpha_T$




Taking the derivatives of Equations 1-4 and 6-9 yields:

**Table C1: Derivatives – Part 1**

| Derivative | $\boldsymbol{\beta_T \leq \alpha_T}$ | $\boldsymbol{\beta_T > \alpha_T}$ |
|---|---|---|
| $\partial A/\partial \alpha_T$ | $1 - \beta_T$ (positive) | |
| $\partial S/\partial \alpha_T$ | $-(1 + \beta_T - \alpha_T)$ (negative) | $-1$ |
| $\partial P/\partial \alpha_T$ | $\beta_T - \alpha_T$ (negative) | $0$ |
| $\partial CP/\partial \alpha_T$ | $-1$ | |
| $\partial A/\partial \beta_T$ | $-\alpha_T$ (negative) | |
| $\partial S/\partial \beta_T$ | $1 - \alpha_T$ (positive) | $1 - \beta_T$ (positive) |
| $\partial P/\partial \beta_T$ | $-(1 - \alpha_T)$ (negative) | $-(1 - \beta_T)$ (negative) |
| $\partial CP/\partial \beta_T$ | $0$ | |

From Appendix A we know that $\partial\delta^*/\partial g \geq 0$, $\partial\delta^*/\partial c \leq 0$ and $\partial\delta^*/\partial r \geq 0$. Besides, we know that:

(10) $$\frac{\partial \alpha_T}{\partial \delta^*} = \frac{\partial \alpha_T}{\partial \mu^*} > 0$$

(11) $\frac{\partial\beta_T}{\partial\delta^*} = \frac{\partial\beta_T}{\partial\mu^*} < 0$

$\alpha_T$ is only a function of $\delta$ and $\mu$, (which are identical, represented by $\delta^*$), and thus $\frac{\partial\alpha_T}{\partial g} = \frac{\partial\alpha_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial g} > 0$, $\frac{\partial\alpha_T}{\partial c} = \frac{\partial\alpha_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial c} < 0$ and $\frac{\partial\alpha_T}{\partial r} = \frac{\partial\alpha_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial g} > 0$. Similarly, $\frac{\partial\beta_T}{\partial c} = \frac{\partial\beta_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial c} > 0$. For the derivative of $\beta_T$ with respect to *r* and *g*, we use the chain rule to yield: $\frac{d\beta_T}{dr} = \frac{\partial\beta_T}{\partial r} + \frac{\partial\beta_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial r} < 0$ and $\frac{d\beta_T}{dg} = \frac{\partial\beta_T}{\partial g} + \frac{\partial\beta_T}{\partial\delta^*}\cdot\frac{\partial\delta^*}{\partial g} < 0$.

We summarize these relationships in Table C2.

**Table C2: Derivatives – Part 2**

| | **Optimal Satiation ($\boldsymbol{\delta^* = \mu^*}$)** | **Inside market quality threshold ($\boldsymbol{\alpha_T}$)** | **Outside market quality threshold ($\boldsymbol{\beta_T}$)** |
|---|---|---|---|
| Gross profit margins per period (*g*) | $\frac{\partial\delta^*}{\partial g} = \frac{\partial\mu^*}{\partial g} > 0$ | $\frac{\partial\alpha_T}{\partial g} > 0$ | $\frac{\partial\beta_T}{\partial g} < 0$ |
| Cost parameter of designing a game (*c*) | $\frac{\partial\delta^*}{\partial c} = \frac{\partial\mu^*}{\partial c} < 0$ | $\frac{\partial\alpha_T}{\partial c} < 0$ | $\frac{\partial\beta_T}{\partial c} > 0$ |
| Retention probability per period (*r*) | $\frac{\partial\delta^*}{\partial r} = \frac{\partial\mu^*}{\partial r} > 0$ | $\frac{\partial\alpha_T}{\partial r} > 0$ | $\frac{\partial\beta_T}{\partial r} < 0$ |

Clearly the conditions for acquisition in equilibrium is the complementary of the conditions for cross promotion and thus are omitted here for the sake of brevity. Similarly, it is easy to see that the signs of the derivatives with respect to *g* are exactly the same as these derivatives with respect to *r* and thus are omitted.

$\frac{\partial S}{\partial g} = \frac{\partial S}{\partial\alpha_T}\cdot\frac{\partial\alpha_T}{\partial g} + \frac{\partial S}{\partial\beta_T}\cdot\frac{\partial\beta_T}{\partial g} < 0$. This derivative is negative as the two terms are.

$\frac{\partial P}{\partial g} = \frac{\partial P}{\partial\alpha_T}\cdot\frac{\partial\alpha_T}{\partial g} + \frac{\partial P}{\partial\beta_T}\cdot\frac{\partial\beta_T}{\partial g}$. The sign of $\frac{\partial P}{\partial g}$ is undetermined as the two terms have opposite signs.

$\frac{\partial CP}{\partial g} = \frac{\partial CP}{\partial\alpha_T}\cdot\frac{\partial\alpha_T}{\partial g} + \frac{\partial CP}{\partial\beta_T}\cdot\frac{\partial\beta_T}{\partial g} < 0$. This derivative is negative as the two terms are.

$\frac{\partial S}{\partial c} = \frac{\partial S}{\partial\alpha_T}\cdot\frac{\partial\alpha_T}{\partial c} + \frac{\partial S}{\partial\beta_T}\cdot\frac{\partial\beta_T}{\partial c} > 0$ . This derivative is positive as the two terms are.

$\frac{\partial P}{\partial c} = \frac{\partial P}{\partial\alpha_T}\cdot\frac{\partial\alpha_T}{\partial c} + \frac{\partial P}{\partial\beta_T}\cdot\frac{\partial\beta_T}{\partial c}$. The sign of $\frac{\partial P}{\partial c}$ is undetermined as the two terms have opposite signs.

$\frac{\partial CP}{\partial c} = \frac{\partial CP}{\partial \alpha_T} \cdot \frac{\partial \alpha_T}{\partial c} + \frac{\partial CP}{\partial \beta_T} \cdot \frac{\partial \beta_T}{\partial c} > 0$. This derivative is positive as the two terms are.

$\frac{\partial S}{\partial \delta^*} = \frac{\partial S}{\partial \alpha_T} \cdot \frac{\partial \alpha_T}{\partial \delta^*} + \frac{\partial S}{\partial \beta_T} \cdot \frac{\partial \beta_T}{\partial \delta^*} < 0$. This derivative is negative as the two terms are.

$\frac{\partial P}{\partial \delta^*} = \frac{\partial P}{\partial \alpha_T} \cdot \frac{\partial \alpha_T}{\partial \delta^*} + \frac{\partial P}{\partial \beta_T} \cdot \frac{\partial \beta_T}{\partial \delta^*}$. The sign of $\frac{\partial P}{\partial \delta^*}$ is undetermined as the two terms have opposite signs.

$\frac{\partial CP}{\partial \delta^*} = \frac{\partial CP}{\partial \alpha_T} \cdot \frac{\partial \alpha_T}{\partial \delta^*} + \frac{\partial CP}{\partial \beta_T} \cdot \frac{\partial \beta_T}{\partial \delta^*} < 0$. The sign of $\frac{\partial CP}{\partial \delta^*} = \frac{\partial CP}{\partial \mu^*}$ is negative as the two terms are.

Combining the information in Table C1 and Table C2 allows us to determine the likelihood of each outcome (acquisition, purchase/ sale, swap, cross-promotion) as a function of *g*, *r*, and *c*. This is summarized in Table C3 which proves propositions 2 and 3.

**Table C3: Derivatives – Part 3**

| | **Likelihood of Acquisition in Equilibrium** | **Likelihood of Swap in Equilibrium** | **Likelihood of Purchase/ Sale in Equilibrium** | **Likelihood of Cross Promotion in Equilibrium** |
|---|---|---|---|---|
| Gross profit margins per period ($g$) | $\frac{\partial A}{\partial g} > 0$ | $\frac{\partial S}{\partial g} < 0$ | Undetermined | $\frac{\partial CP}{\partial g} < 0$ |
| Cost parameter of designing a game ($c$) | $\frac{\partial A}{\partial c} < 0$ | $\frac{\partial S}{\partial c} > 0$ | Undetermined | $\frac{\partial CP}{\partial c} > 0$ |
| Retention probability per period ($r$) | $\frac{\partial A}{\partial r} > 0$ | $\frac{\partial S}{\partial r} < 0$ | Undetermined | $\frac{\partial CP}{\partial r} < 0$ |
| Optimal Satiation ($\delta^* = \mu^*$) | $\frac{\partial A}{\partial \delta^*} = \frac{\partial A}{\partial \mu^*} > 0$ | $\frac{\partial S}{\partial \delta^*} = \frac{\partial S}{\partial \mu^*} < 0$ | Undetermined | $\frac{\partial CP}{\partial \delta^*} = \frac{\partial CP}{\partial \mu^*} < 0$ |

## **Web Appendix D:** Sensitivity Analysis for Correlated Market Quality and *CAC*

In this Web Appendix we prove that the results of Appendix C hold when the outside market quality and *CAC* are correlated. We start by finding an expression for $\beta_T$. Replacing $CAC = \beta^\gamma$ in $\beta_T = \frac{CAC}{CLV}$ and solving for $\beta$ results in:

(1) $\beta_T = \frac{\beta_T^\gamma}{CLV}$

(2) $\beta_T = \left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}$

Compared to Web Appendix C, the main change comes from the diagonal line in the two figures of Case 1 and Case 2, separating purchase and swap. Note that the equation for this line stems from swap and selling conditions Web Appendix B that translates to the following:

(3) $\alpha CLV = \beta \cdot CLV - CAC + RSV$

(4) $\alpha = \beta - \frac{CAC}{CLV} + \frac{RSV}{CLV}$

Replacing $CAC = \beta^\gamma$ in Equation 4 yields:

(5) $\alpha = \beta - \frac{1}{CLV}\beta^\gamma + \alpha_T$

Equation 5 is an implicit function of $\alpha$ and $\beta$. To understand its shape, we plot this function for some exemplary values of $CLV$, $\gamma$, and $\alpha_T$ We see that this function can be well approximated by a straight line for all values of $\beta < 1$. We need this linear approximation to find the value of: *A* - the likelihood of observing acquisition in equilibrium, *P* - the likelihood of observing purchase/ sale at *PAC* at equilibrium, *S* - the likelihood of observing swap at equilibrium, and *CP* - the likelihood of observing any form of cross-promotion (either purchase/ sale or swap). To find such a linear approximation, it is sufficient to find two points that are part of the function.

Setting $\beta = 1$ leads to $\alpha - 1 + \frac{1}{CLV} - \alpha_T = 0$ or $\alpha = \alpha_T - \frac{1}{CLV} + 1$

Setting $\alpha = \alpha_T$ leads to $-\beta + \frac{1}{CLV}\beta^\gamma = 0$ or $\beta = \left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}} = \beta_T$

This gives the two points $P_1\left(\alpha_T - \frac{1}{CLV} + 1; 1\right)$ and $P_2(\alpha_T; \beta_T)$

A straight line connecting these two points has a slope of:

(6) $Slope = \frac{\beta_T - 1}{\alpha_T - (\alpha_T - \frac{1}{CLV} + 1)}$

(7) $Slope = CLV\frac{1-\beta_T}{CLV-1}$

We obtain the intercept as:

(8) $$Intercept = \beta_T - \alpha_T\ CLV \frac{1-\beta_T}{CLV-1}$$

Which gives the final linear approximation of:

(9) $$\beta = \alpha CLV \frac{1-\beta_T}{CLV-1} + \left(\beta_T - \alpha_T\ CLV \frac{1-\beta_T}{CLV-1}\right)$$

(10) $$\beta = \beta_T + (\alpha - \alpha_T\ )CLV \frac{1-\beta_T}{CLV-1}$$

To test the quality of this approximation, we conducted 100,000 simulations. In each run, we take a random value of $CLV$ (between 0 and 10), $\gamma$ (between -1 and 1), and $\alpha_T$ (between 0 and 1). The average fit of the linear approximation over all simulations is 0.976 (squared correlation between predicted and actual values for all values of $\beta < 0.9999$). In 93.9% of cases, the fit exceeds 0.90. This shows that the linear approximation is very reasonable. Note that we can assume $\beta_T < 1$, since if $\beta_T \geq 1$, we obtain a corner solution of little managerial relevance. From Equation 3, $\beta_T < 1$ implies $\gamma < 1$. Also, note that if $\gamma \leq 1$, then $\beta_T < 1$, and thus from Euqation 3, $CLV > 1$. For the remainder of the analysis, we assume $CLV > 1$.

**<u>Case 1</u>: $RSV \geq 1$**

$RSV \geq 1$ implies that $\alpha_T \geq \frac{1}{CLV}$

Then $CLV - CLV\alpha_T \leq CLV - 1$

$$(1-\alpha_T)\frac{CLV}{CLV-1} \leq 1$$

Multiplying by $1 - \beta_T$ yields:

$$(1-\alpha_T)CLV\frac{1-\beta_T}{CLV-1} \leq 1 - \beta_T$$

$\beta_T + (1-\alpha_T)CLV\frac{1-\beta_T}{CLV-1} \leq 1$, and it is easy to see that:

(11) $A = \alpha_T(1-\beta_T)$ – as before

(12) $$S = \beta_T(1-\alpha_T) + 0.5(1-\alpha_T)\left(\beta_T + (1-\alpha_T)CLV\frac{1-\beta_T}{CLV-1} - \beta_T\right)$$

(13) $$S = \beta_T(1-\alpha_T) + 0.5(1-\alpha_T)^2 CLV\frac{1-\beta_T}{CLV-1}$$

(14) $$P = (1-\alpha_T) - S = (1-\alpha_T) - \beta_T(1-\alpha_T) - 0.5(1-\alpha_T)^2 CLV\frac{1-\beta_T}{CLV-1}$$

(15) $$P = (1-\alpha_T)(1-\beta_T) - 0.5(1-\alpha_T)^2 CLV\frac{1-\beta_T}{CLV-1}$$

(16) $CP = P + S = 1 - \alpha_T$ – as before

**Case 2: $RSV < 1$**

$RSV < 1$ implies that $\alpha_T < \frac{1}{CLV}$

Then $CLV - CLV\alpha_T > CLV - 1$

$$(1-\alpha_T)\frac{CLV}{CLV-1} > 1$$

$$(1-\alpha_T)CLV\frac{1-\beta_T}{CLV-1} > 1-\beta_T$$

$$\beta_T + (1-\alpha_T)CLV\frac{1-\beta_T}{CLV-1} > 1$$

The intersection of the line $\beta_T + (\alpha - \alpha_T\,)CLV\frac{1-\beta_T}{CLV-1}$ with $\beta = 1$ is at the point:

$$\beta_T + (\alpha - \alpha_T\,)CLV\frac{1-\beta_T}{CLV-1} = 1$$

$$(\alpha - \alpha_T\,)CLV\frac{1-\beta_T}{CLV-1} = 1-\beta_T$$

$$(\alpha - \alpha_T\,)\frac{CLV}{CLV-1} = 1$$

$$\alpha = 1 - \frac{1}{CLV} + \alpha_T$$

It is easy to see that:

(17) $A = \alpha_T(1-\beta_T)$ – as before

(18) $P = 0.5\left(1 - \frac{1}{CLV} + \alpha_T - \alpha_T\right)(1-\beta_T) = 0.5(1-\beta_T)\left(1-\frac{1}{CLV}\right)$

(19) $S = (1-\alpha_T) - 0.5(1-\beta_T)\left(1-\frac{1}{CLV}\right)$

(20) $CP = 1 - \alpha_T$ – as before

The main change compared to Appendix C, therefore, stems from the size of *S* and *P*. Note that even in the simple case of Appendix C, the sign of the derivative of *P* with respect to *g*, *c*, *r*, and $\delta$ are undetermined. We thus only look into the derivative of *S* with respect to those parameters.

**Case 1 Sensitivity: $RSV \geq 1$**

We substitute $\alpha_T = \frac{RSV}{CLV}$ and $\beta_T = \left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}$ into Equation 18:

$$(21)\qquad S=\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}\left(1-\frac{RSV}{CLV}\right)+0.5\left(1-\frac{RSV}{CLV}\right)^{2}CLV\,\frac{1-\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}}{CLV-1}$$

We substitute $RSV=(\delta\mu)^{T}CLV$:

$$(22)\qquad S=\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}[1-(\delta\mu)^{T}]+0.5[1-(\delta\mu)^{T}]^{2}\frac{CLV}{CLV-1}\left[1-\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}\right]$$

We substitute $\frac{1}{CLV}=\frac{1+d-\delta\mu r}{rg}$:

$$(23)\qquad S=\left(\frac{1+d-\delta\mu r}{rg}\right)^{\frac{1}{1-\gamma}}[1-(\delta\mu)^{T}]+0.5[1-(\delta\mu)^{T}]^{2}\frac{\frac{rg}{1+d-\delta\mu r}}{\frac{rg}{1+d-\delta\mu r}-1}\left[1-\left(\frac{1+d-\delta\mu r}{rg}\right)^{\frac{1}{1-\gamma}}\right]$$

Since $\delta^{*}=\mu^{*}$, we can replace $\delta\mu$ by $\delta^{2}$, which gives:

$$(24)\qquad S=\left(\frac{1+d-\delta^{2}r}{rg}\right)^{\frac{1}{1-\gamma}}(1-\delta^{2T})+\frac{(1-\delta^{2T})^{2}}{2}\frac{rg}{rg-1-d+\delta^{2}r}\left[1-\left(\frac{1+d-\delta^{2}r}{rg}\right)^{\frac{1}{1-\gamma}}\right]$$

Regarding the sensitivity of *S* with respect to the gross profit margins per period (*g*), cost parameter of designing a game (*c*), retention probability per period (*r*), and optimal satiation ($\delta^{*}=\mu^{*}$), we use numerical derivatives. For this, we first draw one million values for each parameter *g*, *c*, *r*, γ, and *d* from a uniform distribution between 0 and .99. We then determine the optimal satiation for this parameter set and the size of *S*. We then repeat these four times, replacing *g* by $g+0.01$, *c* by $c+0.01$, *r* by $r+0.01$, and $\delta$ by $\delta+0.01$. We see that the derivative consistently increases with *g*, *r*, and $\delta$ and consistently decreases with *c*. Hence $\partial S/\partial g>0$, $\partial S/\partial r>0$, $\partial S/\partial\delta>0$, and $\partial S/\partial C<0$.

**<u>Case 2 Sensitivity</u>: $RSV<1$**

We substitute $\alpha_{T}=\frac{RSV}{CLV}$ and $\beta_{T}=\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}$ into Equation 19:

$$(25)\qquad S=\left(1-\frac{RSV}{CLV}\right)-0.5\left[1-\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}\right]\left(1-\frac{1}{CLV}\right)$$

We substitute $RSV=(\delta\mu)^{T}CLV$:

$$(26)\qquad S=1-(\delta\mu)^{T}-0.5\left[1-\left(\frac{1}{CLV}\right)^{\frac{1}{1-\gamma}}\right]\left(1-\frac{1}{CLV}\right)$$

We substitute $\frac{1}{CLV}=\frac{1+d-\delta\mu r}{rg}$:

$$(27)\qquad S = 1-(\delta\mu)^T - 0.5\left[1-\left(\frac{1+d-\delta\mu r}{rg}\right)^{\frac{1}{1-\gamma}}\right]\left(1-\frac{1+d-\delta\mu r}{rg}\right)$$

Since $\delta^* = \mu^*$, we can replace $\delta\mu$ by $\delta^2$, which gives:

$$(28)\qquad S = 1-\delta^{2T} - 0.5\left[1-\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}}\right]\left(1-\frac{1+d-\delta^2 r}{rg}\right)$$

From Equation 28, we obtain:

$$(29)\qquad \frac{\partial S}{\partial g} = -\frac{1}{2(1-\gamma)}\left(1-\frac{1+d-\delta^2 r}{rg}\right)\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}-1}\left(\frac{1+d-\delta^2 r}{rg^2}+\frac{2\delta}{g}\frac{\partial\delta}{\partial g}\right)$$
$$-\frac{1}{2}\cdot\left[1-\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}}\right]\left(\frac{1+d-\delta^2 r}{rg^2}+\frac{2\delta}{g}\frac{\partial\delta}{\partial g}\right)-2T\delta^{2T-1}\frac{\partial\delta}{\partial g}$$

There are three terms in Equation 29, all negative. To see this, we first have to show that

$$(30)\quad \left(1-\frac{1+d-\delta^2 r}{rg}\right)\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}-1}\left(\frac{1+d-\delta^2 r}{rg^2}+\frac{2\delta}{g}\frac{\partial\delta}{\partial g}\right)\geq 0$$

This is true since $\left(1-\frac{1+d-\delta^2 r}{rg}\right) = (1-\frac{1}{CLV}) \geq 0$ since $CLV \geq 1$, similarly the seoncd term in Equation 30 is positive (because $\delta \leq 1$ and $r \leq 1$) and so is the third term since $\partial\delta/\partial g \geq 0$. Similar arguments show that the second term in Equation 30 is negative, and the third term is because $\partial\delta/\partial g \geq 0$. Thus, we have shown that $\partial S/\partial g < 0$.

$$(31)\qquad \frac{\partial S}{\partial c} = -\frac{1}{g(1-\gamma)}\delta\frac{\partial\delta}{\partial c}\left(1-\frac{1+d-\delta^2 r}{rg}\right)\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}-1}-\frac{\delta}{g}\frac{\partial\delta}{\partial c}\left[1-\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}}\right]-$$
$$2T\delta^{2T-1}\frac{\partial\delta}{\partial c}$$

Just as the previous analysis, $\partial S/\partial c$ contains three terms, all positive since $\partial\delta/\partial c \leq 0$ (see Appendix A), and thus we have replicated the same result in Appendix C, that is $\partial S/\partial c > 0$.

$$(32)\qquad \frac{\partial S}{\partial r} = -\frac{1}{2(1-\gamma)}\left(1-\frac{1+d-\delta^2 r}{rg}\right)\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}-1}\left(\frac{2r\delta\frac{\partial\delta}{\partial r}+\delta^2}{gr}+\frac{1+d-\delta^2 r}{r^2 g}\right)-\frac{1}{2}\left[1-\right.$$
$$\left.\left(\frac{1+d-\delta^2 r}{rg}\right)^{\frac{1}{1-\gamma}}\right]\left(\frac{1+d-\delta^2 r}{r^2 g}+\frac{2r\delta\frac{\partial\delta}{\partial r}+\delta^2}{gr}\right)-2T\delta^{2T-1}\frac{\partial\delta}{\partial r}$$

As before, $\partial S/\partial r$ contain three terms, all negative. Thus, we have shown that for Case 2, $\partial S/\partial c > 0$, while $\partial S/\partial g < 0$, and $\partial S/\partial r < 0$.