Research Joint Ventures and R&D Cartels

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We analyze the effects of R&D cartelization and research joint ventures on firms that engage in either Cournot or Bertrand competition in their product market. Research efforts, which precede production, are directed to reducing unit cost and are subject to various degrees of spillovers. It is shown that creating a competitive research joint venture reduces the equilibrium level of technological improvement and increases equilibrium prices compared to when firms conduct R&D independently. A research joint venture that cooperates in its R&D decisions yields the highest consumer plus producer surplus under Cournot competition and, in most cases, under Bertrand competition. (JEL O31)

Forty years ago John Kenneth Galbraith (1952 pp. 91–2) claimed that the era of cheap invention was over and that “Because development is costly, it follows that it can be carried out only by a firm that has the resources associated with considerable size.” In the more recent past it has been asserted that even very large firms do not have adequate resources to undertake unilateral development of some new technologies and, therefore, that numbers of them should conduct development jointly. Sematech, Inc., a consortium of 14 firms, formed to develop new technologies for the production of computer chips, is a prominent example of such a research joint venture (RJV). The alleged advantage of a research joint venture, aside from enabling the participants to overcome a cost-of-development barrier impenetrable to any one of them alone, is the elimination of duplication of effort. Thus, even if each firm in the joint venture were to contribute less than it would spend unilaterally on development, the collective effort might result in the development of the technology at a lower total cost or one superior to what could be achieved by individual efforts. Against these alleged advantages lurk the fears that the participating firms in an RJV will tend to “free ride” on each other, or curtail competition in other phases of their interaction. Indeed, the worst possible case would occur if the research joint venture led to reduced development as compared to noncooperative R&D and product prices rose as a result of the participants’ curtailing competition. The means by which an RJV could enforce price collusion among participants may be quite subtle and are beyond the scope of our present analysis. The issue, then, is how to achieve the alleged advantages of an RJV while avoiding the potential disadvantages. An obvious possible solution is to allow cooperative research while barring any curtailment of competition in product sales. However, this solution leaves open the question of whether or not the firms participating in the RJV should be allowed, by coordinating their R&D decisions, to take

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fully into account the effect of their research and development efforts on their collective profits from the sale of their products. A conventional view might be that the more competition the better and, therefore, that firms should not be allowed to coordinate their R&D decisions.

We address these questions by analyzing four alternative scenarios. Each is modeled as a two-stage noncooperative game with \( n \) firms participating as players. In the second stage of our games the firms are assumed to engage in either Cournot or Bertrand competition, while in the first stage they invest in R&D. A player’s payoff consists of the second-stage production profits less his first-stage R&D expenditure. Each firm employs an identical constant-unit-cost technology in the production of its product in the second stage. Our analysis allows for firms providing differentiated or identical products that share a common production technology. The system of demand functions is symmetric and linear. First-stage research efforts are directed to reducing unit costs. We assume that the function governing cost reduction is concave in R&D expenditure, and we allow spillover from each firm’s research and development efforts to the others. However, in our first two models we suppose that there is no deliberate sharing of the R&D outcomes among the firms, while in the other two we assume that the results of the R&D stage are completely shared, allowing for total elimination of duplication. That is, the first two models describe unilateral R&D activity, while the last two describe RJV’s. Our first model is of R&D competition, in which firms decide on their R&D investments unilaterally, so as to maximize their individual profits. Each firm chooses its R&D investment level given the other’s R&D expenditure while taking into account its resulting second-stage production profits. In our second model, R&D cartelization, firms coordinate their R&D investments, while maintaining competition in the production stage, so as to maximize the sum of their profits. The critical feature of R&D cartelization is that the internal spillover effect does not change when firms coordinate their R&D activity from what it is in the absence of coordination; hence duplication is not eliminated. Our third model is of RJV competition. Here firms operate as in the R&D competition model, except that the results of their R&D are fully shared. Thus, the spillover rate is at its maximal possible level in this case. The last model is of an RJV cartel, in which firms form an RJV, that is, share their information completely, eliminate duplication of effort, and coordinate their R&D expenditure so as to maximize the sum of their profits. Naturally, participants in an RJV could withhold information or be allowed to carry out some propriety research and hence might not necessarily increase the spillover rate to its maximal possible level, as we assume. In this respect, our analysis is confined to the case in which firms share all their R&D outcomes. However, most of our results will continue to hold if the RJV implies a limited increase in the spillover rate (see Section III for a discussion of this issue). When firms cooperate in the R&D stage, there is a reasonable chance that some cooperation in the production stage might develop as well. We assume that such unwarranted cooperation (see the discussion in Section III) can effectively be avoided. Hence, in all four cases considered, firms are not allowed to cooperate in the production stage. The four cases are summarized in Table 1.

Analysis of pure-strategy subgame-perfect Nash equilibria discloses that for sufficiently high spillover rates the reduction in unit costs is greater under R&D cartelization than under R&D competition. The causes of this difference are two types of externalities created by R&D activities in the presence of spillovers. One affects a firm’s competitive advantage relative to its rivals, while the other affects overall industry profits. The two types of externalities are as follows:

(i) Competitive-advantage externality.—A firm deciding on its R&D investment level takes into account the effect it will have on its competitors’ efficiency. In particular, it realizes that if for every dollar it spends on R&D some spills over to its competitors, reducing their
unit costs, it makes them tougher competitors. This externality inhibits a firm's R&D spending. It is taken into account by the firms in all our models.

(ii) Combined-profits externality.—This externality, which can be either positive or negative, is the one conferred by a firm's R&D expenditure on the profits of all the firms. This externality is ignored by a firm under R&D competition but is internalized when firms choose how much to spend so as to maximize an R&D cartel's combined profits.

The total effect of the two externalities is positive when the spillover rate is "sufficiently" high. In this case, unit costs tend to decline more with R&D cartelization than with R&D competition, combined profits are higher, and second-stage equilibrium prices are lower. Thus, both producers and consumers benefit as a result of this type of cooperation among the collaborators in the R&D cartel. The same effect is obtained when comparing RJV competition to RJV cartelization, the latter being the more socially preferable operating mode.

Another way to motivate our results is by recalling that large spillovers (either in the presence or absence of an RJV) create a public-good effect, through elimination of duplication and, hence, some implicit economies of scale that make coordinated actions socially preferable to unilateral ones. Indeed, in the RJV competition scenario these economies of scale are not exploited, and the "free-rider" problem dominates. Thus, among all four models, uncoordinated RJV activity results in the least reduction in per unit cost and the highest product prices. On the other hand, the RJV cartel dominates the other three modes (always under Cournot competition and in most cases under Bertrand competition), as it yields the highest producers' profit and lowest product prices. This, of course, implies that it also achieves the highest total consumer plus producer surplus among the four possible scenarios. Thus, while the antitrust authorities may seek to prevent collusion among the participants in the RJV in the sale of their final product, they may tolerate or even encourage a high degree of coordination in the conduct of research ac-
tivity. The caveat here, of course, is that price collusion among the participants should be effectively prohibited.

Our analysis builds on the model introduced by Claude d'Aspremont and Alexis Jacquemin (1988). In their pioneering approach, they considered a two-stage game: in the first stage, two identical firms conduct research leading to a reduction in unit cost, and the firms are Cournot competitors in the second stage. Their analysis is couched in terms of homogeneous products, and it posits that reduction in unit cost is governed by a quadratic cost function. The focus of their analysis is on the comparison of the magnitude of cost-reducing technical advance achieved when firms conduct R&D competitively versus cooperatively, in the presence of spillover effects. That is, in the former case a firm chooses its R&D expenditure level taking its second-stage profits into account, while in the latter the firms maximize their joint profits. They find that for substantial spillover effects cooperative R&D leads to greater technological advance than competitive R&D. (Similar results were obtained by Michael Spence [1984] and Michael Katz [1986] employing different formulations to address the same issues.)

Even when firms conduct R&D cooperatively, d'Aspremont and Jacquemin suppose that information-sharing remains the same as when it is conducted competitively (i.e., the spillover effect is the same in either case). It is with regard to this assumption that our analysis departs from theirs. Specifically, we distinguish between the possibility of R&D coordination, information-sharing, or both. When information is shared by the participants, the spillover effect increases and an RJV is formed. Our last two models address this information-sharing issue.

We also extend the d'Aspremont and Jacquemin model to more firms than two, a general concave R&D production function, differentiated products, and Bertrand price competition in the product market. These extensions reveal that the sensitivity of d'Aspremont and Jacquemin's results to whether the spillover rate is greater or smaller than a certain critical value carries over with some modifications to the more general models. Our findings also lead to the intuitive economic explanation for why this is so.

There have been a substantial number of extensions to the d'Aspremont and Jacquemin model. Jay Pil Choi (1989) analyzes variants of a two-firm model in which the development of an innovation is an uncertain event, but with the probability of successful innovation an increasing function of the level of resources devoted to R&D. In his first model, the firms compete directly in the same product market. The spillover effect enters this model through the ease with which each firm can imitate the successful one's innovation. Thus, the spillover affects the appropriation and not the innovation process. Choi shows that for spillover effects above a particular level, it is advantageous for the firms to form an RJV. In his second model, Choi assumes that the two firms do not compete in the same product market but share a common technology. Again, the spillover effect enters through the case with which the losing firm can appropriate the winning firm's innovation. This degree of appropriation serves as the losing firm's fallback position in its bargaining with the winning firm over the terms of a licensing agreement.

In other related papers, Raymond De Bondt and Reinhold Veugeler (1991) provide a general summary of strategic investment with spillovers. Kotaro Suzumura (1989) extends d'Aspremont and Jacquemin's results to the case of more than two firms and more general demand and cost assumptions. Nicholas S. Vonortas (1989) extends the d'Aspremont and Jacquemin model to a three-stage game, the first stage of which involves generic R&D, followed by a development stage, and then Cournot competition in the game's third stage. Neil Gandal and Suzanne Scotchmer (1989) address the question of how an RJV might overcome the potential misallocation of resources to R&D (over- or underinvestment) displayed in patent-race models. De Bondt et al. (1992) study the role of the degree of spillover and the number of rivals on total

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R&D activity. John Beath et al. (1990) extend the d’Aspremont and Jacquemin model by dividing the R&D process into two stages. In the first stage, firms invest resources to generate new knowledge, while in the second stage the new knowledge is employed to reduce production costs. Spillovers among the firms can occur at either stage. Beath et al. emphasize that the advantage of an RJV over individual R&D activity arises from sharing of how the new knowledge can be employed to reduce costs. Irene Henriques (1990) points out the role of the magnitude of the spillover parameter on the stability of the equilibrium in the d’Aspremont and Jacquemin R&D investment subgame. In particular, it is shown that the equilibrium is unstable for low levels of spillover.

In Section I our Cournot models are outlined, and in Section II, they are analyzed. The extensions of our models and results to the Bertrand case are briefly discussed in Section III.

I. The Models

We posit n firms each producing a single good. The quantity firm i produces and the price it realizes are denoted by \( Q_i \) and \( P_i \), respectively. The inverse demand function faced by firm i is of the form

\[
P_i = a - Q_i - \gamma \sum_{j \neq i} Q_j
\]

where \( a \) is the demand intercept and \( 0 \leq \gamma \leq 1 \) is the substitutability parameter. Note that all goods are perfect substitutes when \( \gamma = 1 \), and each firm is a monopoly if \( \gamma = 0 \). Initially, each firm operates with the same constant-unit-cost technology, and there is no fixed cost [i.e., if firm i produces quantity \( Q_i \), then its production cost is \( C(Q_i) = cQ_i \)]. We assume that entry into the industry is unprofitable and that \( a > c \) holds, to facilitate profitable production. We let \( \mathbb{N} = \{1, \ldots, n\} \) denote the set of firms.

In all four games considered, there are two stages. In a first stage, every firm \( i \in \mathbb{N} \) simultaneously determines its R&D expenditure level \( x_i \). These decisions determine each firm’s second-stage unit cost. In each game’s second stage, firms engage in Cournot competition. The manner in which R&D is conducted in the industry and the players’ objectives differ across the games considered. In two cases we characterize properties of subgame-perfect Nash equilibria (SPNE) stemming from uncoordinated R&D behavior. In these cases each firm maximizes its individual second-stage production profits net of its first-stage R&D expenditures. In the other two games, firms coordinate their R&D decisions in the first stage so as to maximize combined industry production profits net of total R&D investments. However, in all cases considered, we posit that firms act independently in the game’s second (production) stage. (See Akihiko Matsui [1989] and Section III below for a discussion of what happens when this restriction is relaxed.)

We now present a detailed description of the models. Following the R&D investment stage, the magnitude of unit cost reduction realized by firm \( i \in \mathbb{N} \) is \( f(X_i) \), where \( f \) is an R&D production function, and \( X_i, i \in \mathbb{N} \) is firm i’s effective R&D investment, that is, the amount of money it alone would have had to invest in R&D, if no other firm invested in R&D, to achieve the same unit cost reduction. The magnitude of effective R&D investment \( X_i \) is determined by the combined individual R&D expenditure decisions of all firms and a spillover parameter \( 0 \leq \beta \leq 1 \). The effect of the spillover parameter is that if firm \( j \in \mathbb{N} \) spends \( x_j \) on R&D, then the effect on firm i is as if it had spent \( \beta x_j \). Thus, if the firms invest \( x_1, \ldots, x_n \) in R&D, respectively, then firm i’s effective R&D investment is

\[
X_i = x_i + \beta \sum_{j \neq i} x_j \quad i \in \mathbb{N}.
\]

We say that a research joint venture (RJV) is formed if firms pool their R&D efforts so as to fully internalize the spillover effects (i.e., \( \beta = 1 \) for participating firms). Consequently, when the firms form an RJV, each one’s effective R&D investment is given by

\[
X_i = \sum_{j \in \mathbb{N}} x_j \quad i \in \mathbb{N}.
\]
In all cases considered, unit cost of firm \( i \in \mathbb{N} \) in the production stage is \( c - f(X_i) \).

The R&D process underlying expressions (2) and (3) is supposed to involve trial and error. Put another way, it is a multidimensional heuristic rather than a one-dimensional algorithmic process. The individual firm’s R&D activity does not involve following a single path. If this were the case, the only spillover potential would be from the firm that had somehow forged ahead in the execution of the algorithm to the laggards. However, in an R&D process involving many possible paths and trial and error, it is unlikely that individual firms will pursue identical activities. Indeed, it is reasonable for each firm to pursue several avenues simultaneously, the differences among the firms being in the greater emphasis each places on one over the others. The spillover effect in this vision of the R&D process takes the form of each firm learning something about the others’ experiences; which approaches appear more or less promising and which ones are “dead ends.” This information, which may become available through deliberate disclosure or leak out involuntarily (e.g., at scientific conferences), enables a firm to improve the efficiency of its R&D process by concentrating on the more promising approaches and avoiding the others. In the case of complete information-sharing by the firms, duplication of efforts is avoided, and less promising approaches are weeded out more rapidly than with incomplete sharing. The difference between R&D carried out by the industries’ firms unilaterally and collectively has the flavor of the difference between problem-solving by a computer that operates serially versus one that does parallel processing. When the industries’ firms conduct their R&D privately, each must go through essentially the same entire trial and error process, although not in the same sequence. On the other hand, when they share information completely the R&D process can be divided up into small bits so that the cost of duplicating fruitful and fruitless approaches is avoided.

**Assumption 1:** The R&D production function \( f(X) \) is twice differentiable and concave, \( f(0) = 0 \) and \( f(X) \leq c, f'(X) > 0 \) for all \( X \geq 0 \).

**Assumption 2:** The R&D production function \( f(X) \) satisfies

\[
\lim_{x \to \infty} f(x) < \frac{a-c}{n-1}
\]

and

\[
f'(0) > \frac{(n+1)^2}{2(a-c)}.
\]

While Assumption 1 incorporates some fundamental properties of the R&D production function, Assumption 2 is needed to guarantee that all participating firms find it best to produce actively at the game’s production stage [condition (i) of Assumption 2] and to invest in R&D in the game’s first stage [condition (ii)]. Note that condition (i) of Assumption 2 and the concavity of \( f \) imply

\[
\lim_{x \to \infty} f'(X) = 0.
\]

This property serves to guarantee existence of equilibria in which R&D investment decisions are bounded from above. To establish uniqueness of symmetric equilibria, we also need an assumption that guarantees some unimodality properties of the profit functions. Consider the case of a monopoly. Its overall profits (monopoly profit minus R&D expenditure), expressed as a function of its R&D decision \( X \), would be

\[
[a - c + f(X)]^2/4 - X.
\]

**Assumption 3:** Monopoly profit minus R&D expenditure is a strictly concave func-

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1In this paper we assume the existence of symmetric equilibria and consider only them. Our assumptions serve to guarantee that, whenever there is a symmetric equilibrium, it will be unique. The possibilities of cases in which only nonsymmetric equilibria exist, in one of our models (model C) and in d'Aspremont and Jacquemin's second model, are pointed out by Stephen W. Salant and Greg Shaffer (1992).
tion for \( X \geq 0 \), or equivalently, its derivative and hence the function \( [a - c + f(X)]f'(X) \) decreases in \( X \) for \( X \geq 0 \).  

Let us now describe our four models.  Since the second stage in all cases involves Cournot competition given the technologies acquired in the first stage, we recall that, given the unit cost \( c - f(X_i) \) available to firm \( i \in \mathbb{N} \), its overall profit is given by

\[
\pi_i = Q_i^2 - x_i
\]

where

\[
Q_i = \frac{a - c + f(X_i) - \gamma}{2 + \gamma(n - 1)} \sum_{j \neq i} [f(X_i) - f(X_j)]
\]

and \( X_j, j \in \mathbb{N} \) is firm \( i \)'s effective R&D investment, given by either (2) or (3). The derivation of (5) and (6), as well as establishing that all firms actively produce in this stage, is given in the Appendix. The four cases considered are as follows.

**R&D Competition** (Case N).—This is an extended version of d’Aspremont and Jacquemin’s first model, namely, firms do not create an RJV and do not coordinate their R&D decisions. Thus, each firm simultaneously chooses its R&D investment to maximize its own production profit net of R&D expenditure; that is, it maximizes (5) with respect to \( x_i \) where \( Q_i \) is given by (6) and \( X_j \) is given by (2). An SPNE is determined by simultaneously solving

\[
\max_{x_i} \pi_i \quad \forall \ i \in \mathbb{N}.
\]

The first-order necessary conditions for an SPNE are as follows (note that \( \partial X_i / \partial x_j = 1, j \neq i \):

\[
\frac{\partial \pi_i}{\partial x_i} = \frac{2Q_i}{[2 + \gamma(n - 1)](2 - \gamma)} 
\times 
\left[ (2 - 2\gamma + \gamma n) f'(X_i) - \beta \gamma \sum_{j \neq i} f'(X_j) \right]^{-1}
\]

\[
= 0 \quad i \in \mathbb{N}.
\]

Note that, for \( \gamma > 0 \), the term \( \beta \gamma \sum_{j \neq i} f'(X_j) \) represents the negative externality that the firm’s own R&D effort has on its profits through reducing the other firms’ marginal costs. Under profit unimodality, this externality acts to reduce R&D expenditure. Obviously, when \( \gamma = 0 \) there is no negative externality, as the individual monopolists do not compete. We consider symmetric solutions only, that is, \( X_i = X^N \) (\( N \) stands for noncooperative Nash) for all \( i \in \mathbb{N} \). From (8) and (6) we obtain

\[
\frac{2[a - c + f(X^N)]}{[2 + \gamma(n - 1)]^2(2 - \gamma)} \times [2 - \gamma + \gamma(n - 1)(1 - \beta)] = 1
\]

as the equation determining the equilibrium effective R&D per firm investment \( X^N \). Existence and uniqueness of the solution \( X^N \) to (9) are established in the Appendix.

**R&D Cartelization** (Case C).—This is an extended version of d’Aspremont and Jacquemin’s second model. Here, as in case N, firms do not create an RJV. However, they coordinate their R&D decisions so as to maximize the sum of their combined profits. That is, they form an R&D cartel while maintaining competition in the product markets. The problem solved by the cartel in this case is equivalent, in terms of the first-order condition, to each firm solving

\[
\max_{x_i} T = \sum_{j \in \mathbb{N}} \pi_j
\]

where \( \pi_j \) is given by (5), \( Q_j \) by (6), and \( X_j \) by (2). The first-order necessary optimality conditions for (10) are as shown in
\[
\frac{\partial T}{\partial x_i} = \frac{2}{[2 + \gamma(n-1)](2-\gamma)} \left[ Q \left( 2 - 2\gamma + \gamma n \right) f'(X_i) - \beta \gamma \sum_{j \neq i} f'(X_j) \right] \\
+ \sum_{j \neq i} Q \left[ (2 - 2\gamma + \gamma n) f'(X_j) - \gamma f'(X_i) - \sum_{k \neq j, k \neq i} \gamma \beta f'(X_k) \right]^{-1} \\
= 0 \quad i \in \mathbb{N}
\]
equation (11), above, where \( Q, i \in \mathbb{N} \) are given by (6). Again, assuming symmetry, namely, \( X_i = X^c \) (C stands for collusive) for all \( i \in \mathbb{N} \), and using (11) and (6), one finally obtains the following:

\[
2 \frac{[a - c + f(X^c)]}{[2 + \gamma(n-1)]^2(2-\gamma)} f'(X^c) \\
\times [(2 - \gamma)(1 + \beta(n-1))] = 1.
\]

As in case N, it is possible to verify that our assumptions imply existence and uniqueness of a solution to (12).

Let us now examine the relationship between the first-order necessary conditions if firms choose their research expenditures so as to maximize only their individual profits, condition (8), and compare this to the conditions when they maximize combined profits, condition (11). Note that

\[
\frac{\partial T}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} = \sum_{j \neq i} \frac{\partial \pi_j}{\partial x_i} \quad i \in \mathbb{N}
\]

and that the sum \( \sum_{j \neq i} \frac{\partial \pi_j}{\partial x_i} \) is the combined-profits externality conferred by firm \( i \)'s R&D expenditure on the profits of all the other firms. It is added to the negative competitive-advantage externality that the firm's R&D effort has on its own profit, through reducing the marginal costs of its competitors. It is this externality, positive or negative, that is ignored when each firm chooses its R&D expenditure so as to maximize only its own profit and that is internalized when the firms coordinate their individual R&D expenditures so as to maximize the sum of their profits. Each firm's internalization of the externality its R&D expenditure confers on the other firms is what causes the individual maximization problems in (10) to be equivalent to the joint maximization problem that would be solved by a single director of the cartel.

Now for the symmetric equilibria expressed by (9) and (12) the externality term is

\[
\sum_{j \neq i} \frac{\partial \pi_j}{\partial x_i} = \frac{2[a - c + f(X)]}{[2 + \gamma(n-1)]^2(2-\gamma)} \\
\times f'(X)(n-1)(2\beta - \gamma).
\]

Since the quantity produced by each firm in the second-stage equilibrium, namely, \( [a - c - f(X)]/[2 + \gamma(n-1)] \), is positive, as is \( f'(X) \) by assumption, in an R&D cartel the externality one firm's research expenditure confers upon the others is positive if and only if \( \beta > \gamma/2 \). Note that whenever all the goods are perfect substitutes (\( \gamma = 1 \)), then the above externality is positive if and only if \( \beta > \frac{1}{2} \). If the spillover effect is small, then the unit cost reduction experienced by the other firms as a consequence of an increase in firm \( i \)'s R&D expenditure is not large enough, compared with the unit cost reduction realized by firm \( i \), to avoid reducing their profits. Recall that in a Cournot equilibrium a lower-unit-cost firm's profit rises at the expense of the higher-cost firms. On the other hand, if the spillover effect is sufficiently large (\( \beta > \gamma/2 \)), then all firms' profits rise because total equilibrium profits

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\(^4\) We did not cancel out the term \( 2 - \gamma \) in (12), in order to have the same denominator as in (9). This simplifies the proof of Proposition 1. The same remark applies to expressions (15) and (16).
increase with a reduction in unit costs and the market share of the firms other than firm i does not decline significantly. At $\beta = \gamma / 2$, the opposing effects (of an increase in total equilibrium profits with the decline in unit costs and the loss of profits with a decline in the market share of the higher-cost firms) just balance to make the externality vanish.

**RJV Competition (Case NJ).**—Here firms form an RJV. Thus, the spillover parameter $\beta$ is set to 1 internally. However, as in case N, firms do not coordinate their R&D expenditures. Consequently, each firm simultaneously chooses its R&D investment so as to maximize its own profits given the R&D expenditures of all other firms. Note that the problem solved here is (7) with $\beta = 1$. Hence, letting $\beta = 1$ and $X_i = X_{NI}^j$ (NJ stands for Nash joint venture) for all $i \in \mathbb{N}$ in (8), we get

\[
2 \left[ a - c + f(X_{NI}^j) \right] \left[ 2 + \gamma (n - 1) \right] (2 - \gamma) \times f' (X_{NI}^j) [2 - \gamma] = 1.
\]

\[2 \left[ a - c + f(X_{CI}^j) \right] \left[ 2 + \gamma (n - 1) \right] (2 - \gamma) \times f' (X_{CI}^j) [(2 - \gamma) n] = 1.
\]

**II. Comparison of Models**

A key result in our comparison is the following proposition.

**PROPOSITION 1:** The equilibrium effective R&D investments, $X^\ell$, $\ell = N, C, NJ, CJ$, satisfy the following for all $\beta$:

$X_{CI}^j \geq X_{CI}^N \geq X_{NJ}^N$

Furthermore,

$X_{CI}^N \geq X_{CI}^C$ if and only if $\gamma \leq 2\beta$ holds (equality holding if and only if $\gamma = 2\beta$).

**PROOF:**

Cases N and NJ.—Since

\[2 - \gamma + \gamma (n - 1) (1 - \beta) \leq 2 - \gamma
\]

(equality holding only if $\beta = 1$ or $\gamma = 0$), if $X_{NI}^j$ [the solution to (15)] is substituted for $X^N$ in (9), then the left-hand side in (9) exceeds the right-hand side. Assumption 3 then implies $X_{NI}^j \leq X^N$ (equality if $\beta = 1$ or $\gamma = 0$).

Cases N and CJ.—As above, we compare (16) to (9). Since $2 - \gamma + \gamma (n - 1) (1 - \beta) \leq (2 - \gamma) n$, Assumption 3 implies $X_{CI}^j \geq X^N$ (equality if $\beta = 0$ and $\gamma = 1$).

Cases C and NJ.—Comparing (16) to (12), since $(2 - \gamma) (1 + \beta (n - 1)) \leq (2 - \gamma) n$, $X_{CI}^j \geq X^C$ (equality if $\beta = 1$) follows.

Cases C and NJ.—Comparing (15) to (12), as $(2 - \gamma) [1 + \beta (n - 1)] \geq 2 - \gamma$, $X_{CI}^j \geq X_{NI}^j$ (equality if $\beta = 0$) follows.

Cases N and C.—Comparing (9) to (12), note that $(2 - \gamma) [1 + \beta (n - 1)] \leq 2 - \gamma + \gamma (n - 1) (1 - \beta)$ if and only if $\gamma \leq 2\beta$ (equality holding if and only if $\gamma = 2\beta$). Thus, $X^C \leq X^N$ and only if $\gamma \leq 2\beta$ (equality if and only if $\gamma = 2\beta$).

Note that we have already discussed the intuitive reason for the results of our comparison of cases N and C, that is, the two R&D models. More surprising is the result concerning the two RJV models, NJ and CJ, as the effective R&D expenditure is at its minimum in RJV competition and it attains its maximal level among the four cases considered in an RJV cartel. These results can
be intuitively explained in terms of the "free-rider" effect: in RJV competition, when deciding its R&D investment level, the firm takes into account the large spillover effect it will obtain (and induce) and therefore free rides on other firms' R&D investments. To see this, we show that when firms invest in R&D, say, according to what is prescribed by R&D competition model N, and firm i observes an increase in the value of the spillover parameter $\beta$ then, if all other firms ($j \neq i$) do not change their R&D investments, it finds it best to free ride on its rivals and to decrease its R&D expenditure. Indeed, this will happen if $dx_i/\beta < 0$ holds, $x_i$ being the solution to (8). Differentiating (8) with respect to $\beta$ gives

\[
(17) \quad \frac{d(\partial \pi_i/\partial x_i)}{d\beta} = \frac{\partial(\partial \pi_i/\partial x_i)}{\partial \beta} + \frac{\partial^2 \pi_i}{\partial x_i \partial \beta} \frac{dx_i}{d\beta} = 0.
\]

Under second-order sufficiency in (8), $\partial^2 \pi_i/\partial x_i^2 < 0$. Furthermore, it can be shown that Assumption 3 implies that marginal profitability of R&D investment diminishes as spillover increases, that is,

\[
\partial(\partial \pi_i/\partial x_i)/\partial \beta < 0.
\]

Altogether, these two inequalities imply by (17) that $dx_i/\beta < 0.$ The free-rider effect is totally eliminated in the RJV cartel when firms coordinate their R&D investments, as competition in the R&D stage is curtailed. Hence, the free-rider effect exists in R&D competition (but not in model C), but to a smaller extent as the spillover rate is lower ($\beta < 1$) than in RJV competition. Thus, effective R&D investment under R&D competition is larger than under RJV competition.

Since $f$ is increasing in $X$, we obtain from Proposition 1 the following result characterizing the equilibrium reduction in unit cost.

**COROLLARY 1:** For all $\beta$ the equilibrium technological improvements $f(X)$ satisfy

\[
f(X^{CJ}) \geq f(X^C) \geq f(X^{NJ})
\]

\[
f(X^{CJ}) \geq f(X^N) \geq f(X^{NJ}).
\]

Furthermore, $f(X^C) \geq f(X^N)$ if and only if $\gamma \leq 2\beta$.

Let

\[
Q' = \frac{a - c + f(X')}{2 + \gamma(n-1)}
\]

\[\ell = N, C, NJ, CJ\]

denote the outputs, obtained from (6), produced by each firm in the above four cases, and let $P', \ell = N, C, NJ, CJ$, denote the corresponding equilibrium prices. Corollary 1, implies the following.

**COROLLARY 2:** For all $\beta$, the equilibrium prices, $P', \ell = N, C, NJ, CJ$, satisfy

\[
P^{CJ} \leq P^C \leq P^{NJ}
\]

\[
P^{CJ} \leq P^N \leq P^{NJ}.
\]

Furthermore, $P^C \leq P^N$ if and only if $\gamma \leq 2\beta$.

Note that the results obtained in Proposition 1 and Corollaries 1 and 2 for cases $N$ and $C$ extend those of d'Aspremont and Jacquemin to $n$ firms, $n$ substitute products, and a general convex R&D cost function. We turn now to profit comparisons. Denote by $\pi', \ell = N, C, NJ, CJ$, the individual firm's equilibrium profit in each of the above four cases.

**PROPOSITION 2:** The individual firm's equilibrium profits satisfy

\[
\pi^{CJ} \geq \pi' \quad \ell = N, C, NJ
\]

\[
\pi^C \geq \pi^N.
\]

**PROOF:**

Cases $N$ and $CJ$.—Note that it is feasible, though not necessarily optimal, in case CJ
that each firm spends \( x = X^N/n \) on R&D. Thus, (5) and (18) yield

\[
(19) \quad n\pi_{CI} \geq n \left[ \frac{a - c + f(X^N)}{2 + \gamma(n-1)} \right]^2 - X^N.
\]

However (2) gives

\[
(20) \quad X^N = [1 + \beta(n-1)]x^N
\]

where \( x^N \) denotes the individual firm's equilibrium R&D expenditure when \( \ell = N, C, NJ, CJ \). Since \( 1 + (n-1)\beta \leq n \) (equality if \( \beta = 1 \)), it follows by (20) that \( X^N \leq nx^N \). Thus,

\[
(21) \quad n \left[ \frac{a - c + f(X^N)}{2 + \gamma(n-1)} \right]^2 - X^N \\
\geq n \left[ \frac{a - c + f(x^N)}{2 + \gamma(n-1)} \right]^2 - nx^N \\
= n\pi^N
\]

and \( \pi_{CI} \geq \pi^N \) follows by (19) and (21).

Cases \( C \) and \( CJ \).—Via a similar argument,

\[
(22) \quad n\pi_{CI} \geq n \left[ \frac{a - c + f(X^C)}{2 + \gamma(n-1)} \right]^2 - X^C \\
\geq n \left[ \frac{a - c + f(x^C)}{2 + \gamma(n-1)} \right]^2 - nx^C \\
= n\pi^C
\]

where the second inequality in (22) follows

\[
(23) \quad X^C = [1 + \beta(n-1)]x^C \leq nx^C.
\]

Cases \( NJ \) and \( CJ \).—Similarly,

\[
(24) \quad n\pi_{CI} \geq n \left[ \frac{a - c + f(X^{NJ})}{2 + \gamma(n-1)} \right]^2 - X^{NJ} \\
= n \left[ \frac{a - c + f(x^{NJ})}{2 + \gamma(n-1)} \right]^2 - nx^{NJ} \\
= n\pi^{NJ}.
\]

Cases \( N \) and \( C \).—A feasible, though not necessarily optimal, possibility in case \( C \) is that each firm invests \( X^N/(1 + \beta(n-1)) \) in R&D. Hence,

\[
(25) \quad n\pi^C \geq n \left[ \frac{a - c + f(X^N)}{2 + \gamma(n-1)} \right]^2 \\
- n \frac{X^N}{1 + \beta(n-1)} = n\pi^N
\]

where the last equality in (25) follows (20).

Thus, Corollary 1 implies that among the four cases considered, the lowest technological improvement is obtained under RJV competition (case NJ), and the highest is obtained in an RJV cartel (case CJ). The implications of Corollary 2 and Proposition 2 are summarized in the following theorem, our main result.

THEOREM 1: The equilibrium product prices attained in RJV competition are the highest among all the cases considered, while RJV cartelization yields the highest equilibrium total (consumer plus producer) surplus. Furthermore, an RJV cartel dominates all other cases as it yields the highest per-firm profit and the lowest product prices among all cases considered in equilibrium.

III. Discussion and Extensions

We have compared four possible R&D organization modes (R&D competition, R&D cartelization, RJV competition, and RJV cartelization) and found that RJV competition is the least desirable because it leads to the lowest equilibrium technological improvement and the highest product price. On the other hand, RJV cartelization was found to be the most desirable because it yields both the highest firms' profits and lowest product prices. Moreover, from Proposition 1 it follows that, if R&D requires a minimum investment in order to be effective at all, then there may be cases in which it will be undertaken by an RJV but not by the firms acting individually.
The policy implications of our results are obvious. In markets that coincide with our suppositions, an RJV in its competitive form is undesirable. However, firms should be encouraged to form RJV's only if they coordinate their R&D (but not product output) decisions, that is, only if they form an RJV cartel. This is the seemingly counterintuitive result of our analysis for it might seem at first glance that maintaining competition among the firms in both R&D and production would be most desirable.

There are three issues that come to mind as possible extensions of our results:

(i) \textit{Allowing the spillover rate }$\beta$\textit{ to increase to a value greater than the initial one but less than the maximal value of 1, in an RJV.}—It can be shown that, under these circumstances, Proposition 2 will continue to hold, while Proposition 1 and its two corollaries will hold with the modification that $X^C > X^N$, and $X^{CI} > X^N$ (and the corresponding inequalities for $f$ and $P$ in Corollaries 1 and 2, respectively) will be satisfied only if $\gamma \leq 2\beta$ holds.

(ii) \textit{Complete cartelization.}—It can be shown that if the firms are allowed to coordinate their production decisions in addition to the RJV cartel, that is, to create a (complete) cartel in both R&D and production decisions, then technological improvement will exceed that of the RJV cartel. However, examples can be constructed in which a complete cartel yields higher product prices than the RJV cartel, even though it results in a higher total consumer-plus-producer surplus.

(iii) \textit{Price competition.}—The question is to what extent our results from quantity-competition models carry over to price-competition models. It can be shown that most of our former results hold in this case for all values of the substitution parameter $\gamma$, while the remainder hold for a somewhat restricted range of $\gamma$. The models we discuss here are the counterparts of the four models (N, C, NJ, and CJ) of Table 1, except that the second-stage games involve Bertrand competition. Here we present only the main features of the analysis. The detailed derivation is available from the authors upon request.

We begin by noting that the system of inverse demand functions (1), yield the demand functions:

\[
Q_i = \hat{a} - V_1 P_i + V_2 \sum_{j \neq i} P_j \quad i \in \mathbb{N}
\]

where

\[
\hat{a} = \frac{a}{(n-1)\gamma + 1}
\]

and

\[
V_1 = \frac{(n-2)\gamma + 1}{((n-1)\gamma + 1)(1-\gamma)}
\]

\[
V_2 = \frac{\gamma}{((n-1)\gamma + 1)(1-\gamma)}.
\]

The counterpart of Proposition 1 is the following.

\textbf{PROPOSITION 1*:} \textit{The equilibrium effective R&D investments in the price-competition games, denoted by }$Y^\ell$, $\ell = N, C, NJ, CJ$, \textit{satisfy}

\[
Y^{CI} > Y^C > Y^{NJ}
\]

\[
Y^N > Y^{NJ}
\]

for all $\beta$ and $\gamma$. Furthermore, $Y^{CI} > Y^N$ if and only if $\gamma \leq \gamma_1(\beta)$, and $Y^C > Y^N$ if and only if $\gamma \leq \gamma_2(\beta)$, where both $\gamma_1(\beta)$ and $\gamma_2(\beta)$ are increasing in $\beta$, satisfying $\gamma_1(\beta) \leq \gamma_2(\beta)$ and $\gamma_1(1) = \gamma_2(1) = 1$.

Note that for both quantity-competition and price-competition models effective R&D investment is greater under R&D cartelization (case C) than under R&D competition (case N) if $\gamma$ is small enough. A
difference between the two models is observed when comparing R&D competition (N) with the RJV cartel (CJ). While under quantity competition effective R&D investment in RJV cartelization is larger than in R&D competition for all values of γ, under price competition this happens only for values of γ below a critical value, given by γ(β). Although this is certainly limiting, it can be shown that γ(β) ≥ 2/3. Thus the counterpart of the result obtained under quantity competition, Y C1 ≥ Y N, holds at least for 0 ≤ γ ≤ 2/3. The intuition for the above difference lies in the observation that when γ = 1 (all goods are perfect substitutes) symmetric firms make zero profit at the equilibrium of the second-stage pricecompetition game. Consequently, it does not pay to participate in an RJV. It should be pointed out though that, for the same reasons, when γ = 1, a symmetric equilibrium in which more than one firm survives cannot exist in the initial R&D-competition (N) price-setting game. More generally, it can be shown that (i) for any given value of the substitution parameter γ, the number of firms that can participate in the original R&D-competition price-setting game is bounded from above, and (ii) when the number of active firms declines, the range of γ over which Y C1 ≥ Y N applies widens. Thus, even for values of γ that are close to 1, the number of firms that are candidates for participation in an RJV will be such that the RJV cartel is more likely to increase the effective R&D investment and decrease prices, while increasing firms' profits.

As with Proposition 1 and quantity-competition models, Proposition 1* is the key result in establishing the relationships among equilibrium outcomes of the four price-competition models. It follows that all other propositions and corollaries, obtained for the quantity-setting games, continue to hold in the price-competition setting, under the same restrictions on γ specified in Proposition 1*. In particular, RJV competition is bound to yield the highest product prices among the four models, while the RJV cartel will always yield the highest total profits as well as the lowest product prices whenever γ is less than γ(β).

**APPENDIX**

_Derivation of Equations (5) and (6)._—By (1), the second-stage profit of firm i, \( g_i(Q) \) is

\[
(A1) \quad g_i(Q) = Q \left( a - Q_i - \gamma \sum_{j \neq i} Q_j - c_i \right)
\]

where \( Q = \{Q_1, \ldots, Q_n\} \) is the outputs vector. A second-stage equilibrium is obtained by solving

\[
(A2) \quad \max_{Q_i} g_i(Q) \quad \forall \ i \in \mathbb{N}.
\]

Assuming \( Q \geq 0 \) for the time being, the first-order necessary condition to (A2) is

\[
(A3) \quad a - Q_i - \gamma \sum_{j \neq i} Q_j - c_i = 0
\]

or

\[
(A4) \quad (\gamma - 2)Q_i - a = \gamma \sum_{j \in \mathbb{N}} Q_j - a
\]

Thus,

\[
(A5) \quad Q_i = \frac{c_i - c_i}{2 - \gamma} + Q_j \quad \forall \ i, j \in \mathbb{N}
\]

follows. Substituting (A5) in (A3) and rearranging,

\[
(A6) \quad Q_i = \frac{c_i - c_i + \gamma \sum_{j \neq i} (c_j - c_i)}{2 + \gamma (n - 1)}
\]

follows. Letting \( c_i = c - f(X_i) \), (6) is obtained. Furthermore, (5) follows from substituting (A3) in (A1).

To establish that all firms actively produce in a second-stage equilibrium, we have to show that \( Q_i \), given by (6), cannot be negative or, following some rearrangements, that

\[
(A7) \quad a - c + \frac{2 - 2 \gamma + \gamma n}{2 - \gamma} f(X_i) - \frac{\gamma}{2 - \gamma} \sum_{j \neq i} f(X_j) > 0
\]
holds. Note that (A7) will hold if it is satisfied when \( X_j = 0 \) and \( X_i \) is at its maximal value for all \( j \), namely, when

\[
(A8) \quad \lim_{X \to -} f(X) < \frac{(a - c)(2 - \gamma)}{\gamma(\gamma - 1)}
\]

holds. As the right-hand side of (A8) is at its lowest level when \( \gamma = 1 \), (A8) indeed holds by condition (i) of Assumption 2.

Existence and Uniqueness of the Solution to Equation (9).—First note that condition (i) of Assumption 2 and equation (4) imply that the left-hand side of (9) is smaller than the right-hand side for sufficiently large \( X \). Hence, to establish existence we need to show that the left-hand side of (9) is greater than the right-hand side at \( X = 0 \), namely,

\[
(A9) \quad \frac{2(a - c)[2 - \gamma + \gamma(\gamma - 1)(\gamma - 1)]}{[2 + \gamma(\gamma - 1)]^2} f''(0) > 1.
\]

The left-hand side of (A9) is at its lowest value when \( \gamma = 1 \). Thus, (A9) holds if

\[
(A10) \quad \frac{2(a - c)}{[2 + \gamma(\gamma - 1)]^2} f''(0) > 1.
\]

As the left-hand side of (A10) is least when \( \gamma = 1 \), the inequality follows by condition (ii) of Assumption 2. Thus, existence of a solution to (9) is established. Uniqueness follows from Assumption 3.

REFERENCES


